TABLE OF CONTENTS

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Patterns and Relationships

Look at the following tables. Do you see a pattern between the words on the left and the words on the right?

Complete the table. Add additional information to the table.

girl	boy
top	bottom
first	
big	

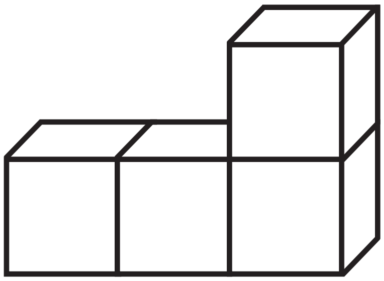
carpenter	hammer
teacher	books
author	computer
doctor	
secretary	

penny	one
nickel	five
dime	
	twenty-five

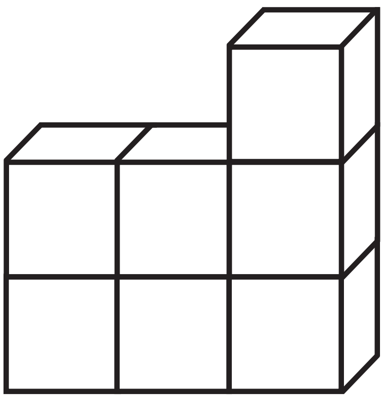
3	5
8	10
0	2
10	
	6

5	10
3	6
10	20
100	
	50

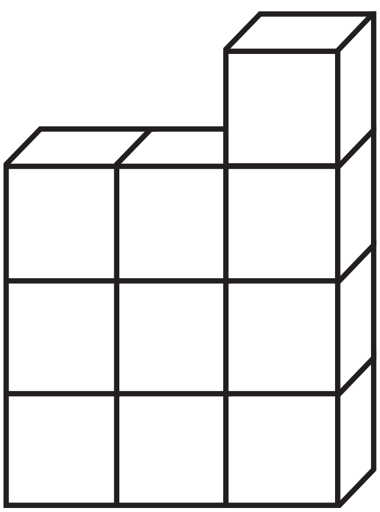
Piles of Cubes I



Pile 1



Pile 2



Pile 3

Pile Number	Number of Cubes
1	4
2	7
3	10
4	
5	
10	

Things I want to remember:

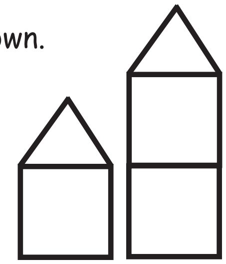
A Menu of Pattern Activities

1. Toothpick Houses

Materials: Flat toothpicks

A one and a two story toothpick house are built as shown.

How many stories can you build if you have 35 toothpicks?



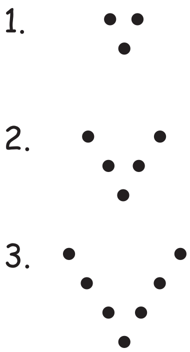
Story Number	Number of Toothpicks
1	6
2	9

2. Flying Vs and Ws

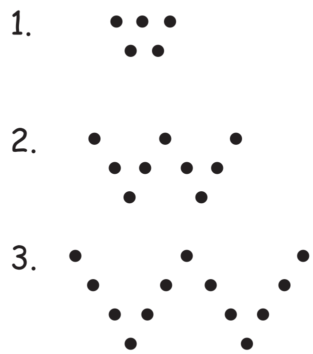
Materials: Cubes, tiles, or similar objects

Birds and airplanes often fly in a shape that forms a **V** or **W**.

- Investigate the number of birds in the **V** shape and the number of airplanes in the **W** shape.
- Find the number of birds and the number of airplanes in the 10th pattern.



Pattern Number	Number of Birds
1	3
2	5
3	

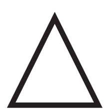


Pattern Number	Number of Airplanes
1	5
2	9
3	

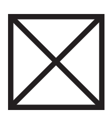
3. Diagonals of Polygons

Materials: Paper and pencil

Use what you know about patterns and relationships to find the number of diagonals in a decagon, a ten-sided figure.



Triangle =
3 sides
0 diagonals



Quadrilateral =
4 sides
2 diagonals

Sides	Diagonals
3	0
4	2
5	
10	

4. Cut-Cut

Materials: Paper and scissors

- Cut a sheet of paper in half and count the pieces.
- Cut each piece in half and count the pieces after the second cut.
- Continue to cut and count.

How many pieces do you have after 5 cuts?
How many would you have if you made 8 cuts?

Number of Cuts	Number of Pieces
1	2
2	4
3	

At Home with Patterns

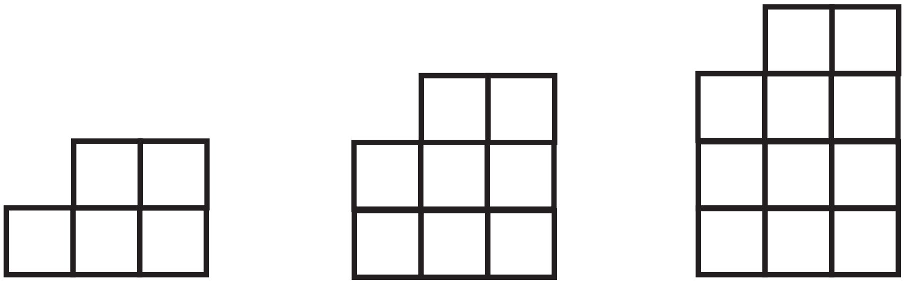
1. Explore patterns with your children at home. Use a table to record them.

2. Find mathematical patterns in your life. Use a table to record them.

3. Choose one of the activities from **A Menu of Pattern Activities** that you did not complete and complete it at home. Make it a family project.

Piles of Cubes II

1. Follow the pattern to build the next three piles.



Pile 1 Pile 2 Pile 3

2. Use the piles and cubes to complete the table.

Pile Number	Number of Cubes
1	5
2	8
3	11
4	
5	
6	

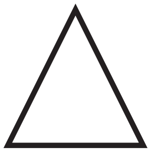
3. Answer the following questions.
   - *How many cubes would there be in the 10th pile?*
   - *The 20th pile?*
   - *How do you know?*

4. Use words to describe how to find the number of cubes in the 100th pile.
   - *If **p** is the pile number, then how many cubes will be in **p** piles?*
   - Think: *What are you doing to the pile number to get the number of cubes?*

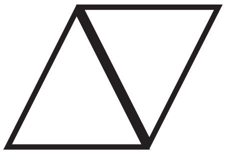
5. Use the rule that your wrote in question number 4 above to find out how many cubes would be in pile 15.

New Terminology:
p is called a variable.
The rule you wrote is called an algebraic expression.

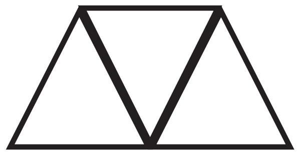
Triangles



The perimeter of one triangle is 3 units.



The perimeter of two triangle is 4 units.



The perimeter of three triangle is 5 units.

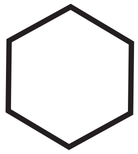
Follow the pattern and complete the table.

Number of Triangles	Perimeter
1	3
2	4
3	5
4	
5	
6	
t	

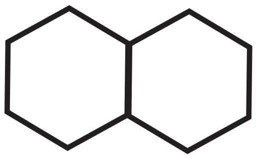
Look: Every time a triangle is added what happens to the perimeter?

Think: Can you write an algebraic expression for **t** number of triangles?

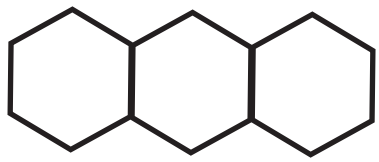
Hexagons



The perimeter of one hexagon is 6 units.



The perimeter of two hexagons is 10 units.



The perimeter of three hexagons is 14 units.

Follow the pattern and complete the table.

Number of Hexagons	Perimeter
1	6
2	10
3	14
4	
5	
6	
h	

Look: Find the pattern. Every time a hexagon is added what happens to the perimeter?

Think: What am I doing to the number of hexagons to find the perimeter?

From Relationships to Expressions

Models ⟶ Tables ⟶ Expressions

How?

- Use a table to record relationships
- Look for a pattern
- Describe the pattern in words
- Assign a letter to the variable
- Convert your words to an algebraic expression

What do you need to know?

- How to set up a table
- How to use variables
- Different ways to write multiplication with variables
- How to write algebraic expressions

Writing Equations

Example: 6 hours = _____ minutes
10 hours = _____ minutes

Calculations:

1. 3 days = _____ hours
To find the number of hours ______________
____

2. 5 quarters = _____ pennies
To find the number of pennies ____________
____

3. 35 days = _____ weeks
To find the number of weeks _____________
____

4. 36 months = _____ years
To find the number of years ______________
____

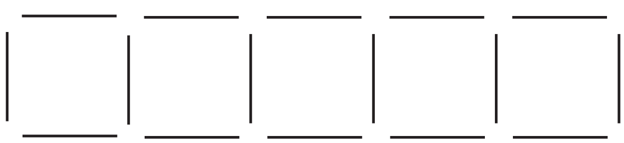
Equations and Problem Solving

Exploring Equations

1. Matchstick Track

A matchstick track has 5 squares.

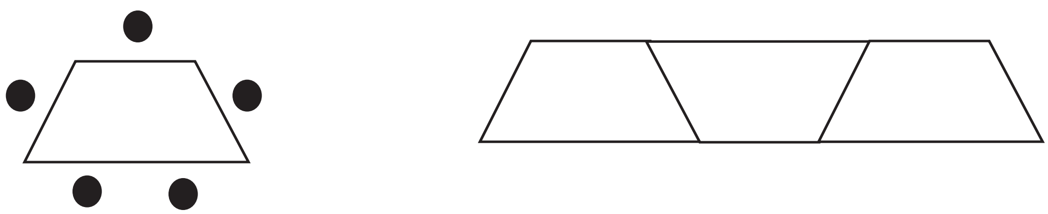
- *If the pattern continues, how many matches would be needed to build a track with 60 squares?*
- *Write an equation for finding the number of matches needed for any number of squares.*



2. Banquet Tables

Tables in the shape of a trapezoid are placed side by side in the following manner. The dots show how many people can sit at a table.

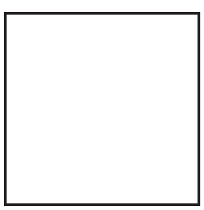
- *How many people can sit when there are 3 tables side by side? 10 tables? 50 tables?*
- *Write an equations for finding the number of people that can sit at any number of tables.*



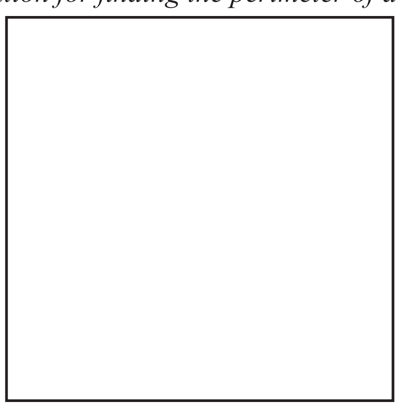
3. Growing Squares

Each of these are squares.

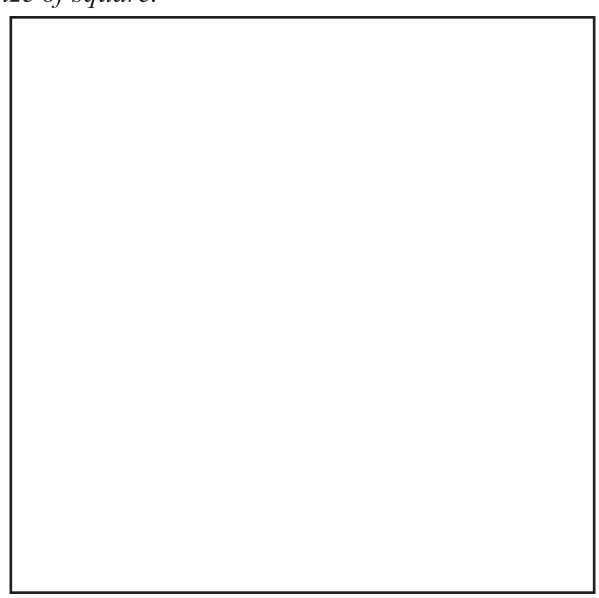
- *As the side of the square grows by one unit, what happens to the perimeter?*
- *Write an equation for finding the perimeter of any size of square.*



1”



2”



3”

Side of Square	Perimeter
1	4
2	
3	

Equations and Problem Solving

Exploring Equations

These problems are more difficult. Find a pattern for solving them.

4. Twelve Days of Christmas

In the song, "The Twelve Days of Christmas" the total number of gifts received is a triangular number.

a) On the first day there was 1 gift
b) On the second day there were 2 new gifts
c) On the third day there were 3 new gifts, etc., until the 12^{th} day.

- *How many gifts were received on the 12^{th} day?*
- *What is the total of gifts received during all 12 days?*
- *Why is this number a triangular number?*

5. Handshakes

One hundred people shake hands.

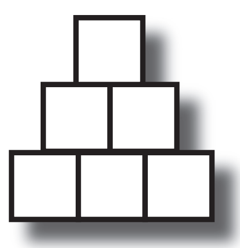
How many handshakes take place?

6. Tower of Offices

The Alamo Construction Company is building a tower like this. Each square represents an office. There will be:

a) One office on the top floor
b) Two offices on the second floor
c) Three on the 3^{rd}, etc.

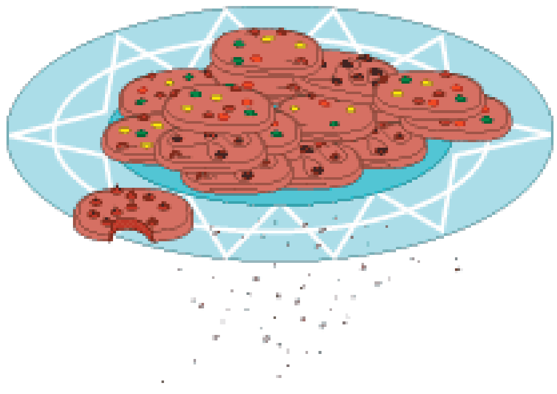
- *Using this plan is it possible to have exactly 34 offices?*
- *If this plan is used to build a tower with 22 floors, how many office will there be?*



Problem Solving Strategies

- look for a pattern
- construct a table
- make an organized list
- act it out
- draw a picture
- use objects
- guess, check and revise
- work backwards
- write an equation
- solve a simpler/similar problem
- make a model
- use logical reasoning
-
-
-
-
-
-
-

Bag of Cookies



There are 24 cookies that need to be put in bags. Each bag will have 3 cookies.

How many bags will there be?

Engaging Your Children in Problem Solving

Most problems can be solved using a variety of strategies. Below the suggested strategies are some problem which you can use with your children.

1. Possible strategies: Draw a picture; Use objects
   a) Alex caught 13 fish. He gave 4 away. How many fish does he have now?

   b) David and Joe ran a 50 meter race. When Joe crossed the finish line, David was 8 meters behind. How far had David fun so far?

   c) If 10 cookies are shared by 8 people so each person gets the same amount of cookies, how many cookies will each person get?

   d) Maria trades in her animals for smaller animals and always gets 4 new animals for each animal she trades in. She traded in her cow for 4 goats, each goat for 4 chickens, and each chicken for 4 birds. How many birds does she have?

2. Possible strategies: Make an organized list, Construct a table

   a) How many different ways can you make 25 cents using pennies, nickels and/or dimes?

   b) I am a number between 40 and 60. I am even. I have a reminder of 3 when divided by 5. I have a reminder of 2 when divided by 7. Who am I?

   c) Sara packed 3 shirts and 3 pants for her trip. Her shirts are red, white and blue. Her pants are red, white and blue. How many different outfits can Sara wear on her trip?

   d) Josh, Samantha, Sonia, and Alex have different color hair. One has red hair; one has brown hair, one has blond hair and one has black hair. One of the boys has brown hair. Josh has blond hair. Alex does not have red hair.

   e) Jerry's grandmother offered to pay him for chores during his stay with her. She gave him two choices on how she could pay him. First choice: he could have $1.00 for the first day, $2.00 the second day, $3.00 the third day and she would continue to give him one more dollar than the previous day for 2 weeks. Second choice: he could get paid 1¢ the first day, 2¢ the second day, 4¢ the third day, and double the amount of the preceding day. Which choice should Jerry take?

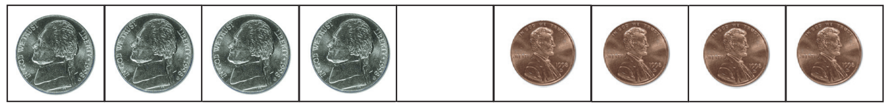
Money Exchange Game

Materials: Pennies and nickels

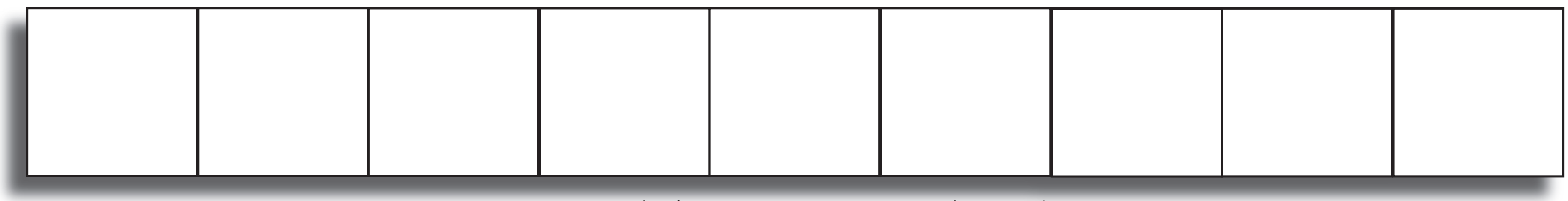
Objective: Move the nickels to the pennies' squares and the pennies to the nickels' squares.

Rules:
1. Move only one coin at a time.
2. Nickels must always move to the right and pennies must always move to the left.
3. A coin can slide to an empty square next to it.
4. A nickel can jump a penny or a penny can jump a nickel if:
   - it jumps a coin next to it, and
   - it lands on an empty space

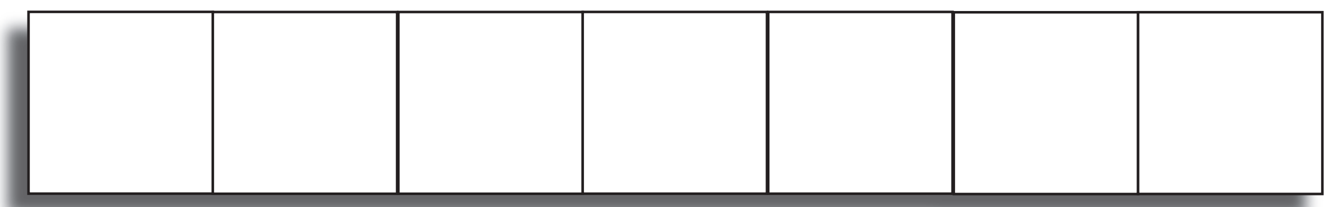
Example:



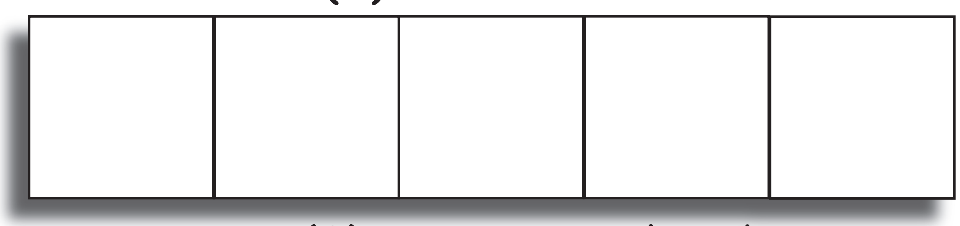
Game Board



Four (4) coins on each side



Three (3) coins on each side



Two (2) coins on each side

Tables and Expression Cards

Set A-F

Copy one set for each group. Copy each set on different colors of cardstock

A

3	5
5	7
11	13
a	?

B

4	3
7	6
1	0
x	?

C

2	2.5
5	5.5
7.5	?
g	?

D

10	40
25	100
3	12
U	?

E

6	3
2	1
10	5
14	?
m	?

F

4	9
8	17
2	5
10	?
1s	?

Tables and Expression Cards

Set G-L

G

6	2
12	4
9	?
m	?

H

2	4
6	36
3	9
h	?

I

4	9
5	10
6	?
g	?

J

100	10
40	4
15	1.5
t	?

K

3	5
4	7
5	9
b	?

L

1	11
3	13
-1	9
?	15
k	?

Tables and Expression Cards

Set M-R

M

N

O

P

Q

R

Tables and Expression Cards

Set S-X

S

T

U

V

W

X

From Tables to Expressions

- Each group has six cards. Pass the cards out, so each person has one.
- Each person in a group works individually on a card.
- Each person completes the table on their card by writing the missing information and the expression on the back and passing it to the person on their right.
- The next person takes the card and determines the missing information, then checks their work with the work on the back.
- If there is agreement pass the card to the right.
- If not, discuss why not before going to the next card.
- Continue until all six cards are completed.

Family Expressions

Symbol	Meaning	Numerical Value
m	Number of men in a family	1 man in a family
w	Number of women in a family	2 women in a family
c	Number of children in a family	3 children in a family
t	Number of cats per family	2 cats per family
d	Number of dogs per family	1 dog per family
n	Number of families in the neighborhood	8 families in the neighborhood
r	Number of cars in a family	2 cars per family
g	Number of gallons of gas a car uses in a day	4 gallons per day
p	Price of one gallon of gasoline	$1.23 per gallon
a	Amount of water an adult drinks in a day	64 ounces
h	Amount of water a child drinks in a day	38 ounces

Family Expressions

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m	Number of men in a family	1 man in a family
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a	Amount of water an adult drinks in a day	64 ounces
h	Amount of water a child drinks in a day	38 ounces

The table above assigns variables for certain quantities. For example, M stands for the number of men in a family. A value for each variable is given. Treat this value as a constant for all cases.

Use the given symbols to write algebraic expressions that make sense. Some expressions you make will have no meaning. For example, P + G indicates that the number of pets and the price of one gallon of gas should be added, but the answer is not meaningful.

NCTM Algebra Standard

Instructional programs from prekindergarten throught grade 12 should enable all students to--

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

NCTM Process Standards

- **Problem Solving**
- **Reasoning & Proof**
- **Communication**
- **Connections**
- **Representation**

At Home with Expressions

Complete the table. Then use the variable to write an expression for each table.

1	5
2	10
3	15
4	
5	
n	

3	5
4	6
8	10
11	
	18
m	

4	9
7	15
10	21
15	
	41
t	

2	1
4	2
5	2.5
8	
	12
s	

Write an algebraic expression. Complete a table for the expression.

At Home with Expressions Answers

Complete the table. Then use the variable to write an expression for each table.

1	5
2	10
3	15
4	20
5	25
n	5n, n*5, or 5 x n (5 times n)

3	5
4	6
8	10
11	13
16	18
m	m + 2, or 2 + m (add 2 to m)

4	9
7	15
10	21
15	31
20	41
t	2t + 1, or t * 2 + 1 (multiply t by 2 and then add 1)

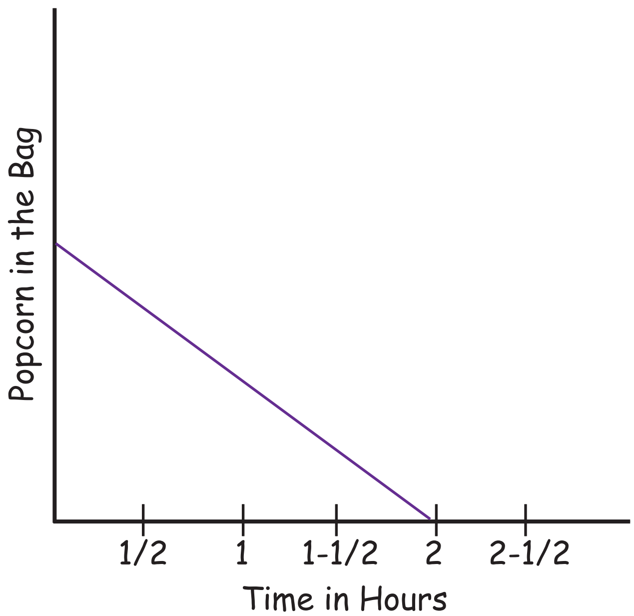
2	1
4	2
5	2.5
8	4
6	12
s	s/2 or s ÷ 2 (divide s by 2)

Write an algebraic expression. Complete a table for the expression.

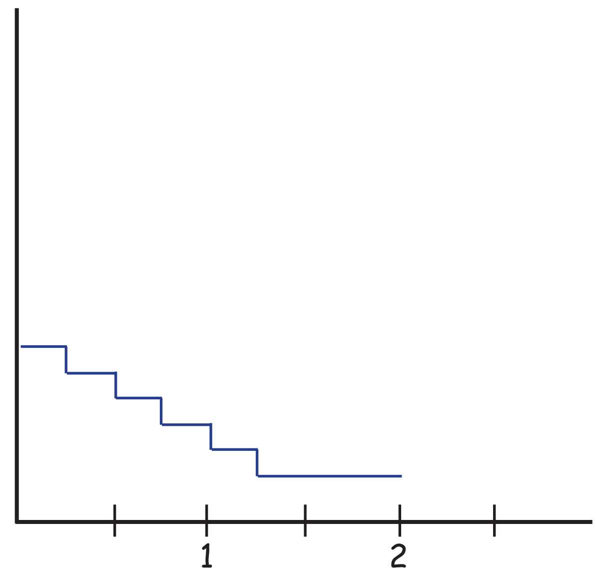
Answers will vary

Popcorn Graphs

Graph 1

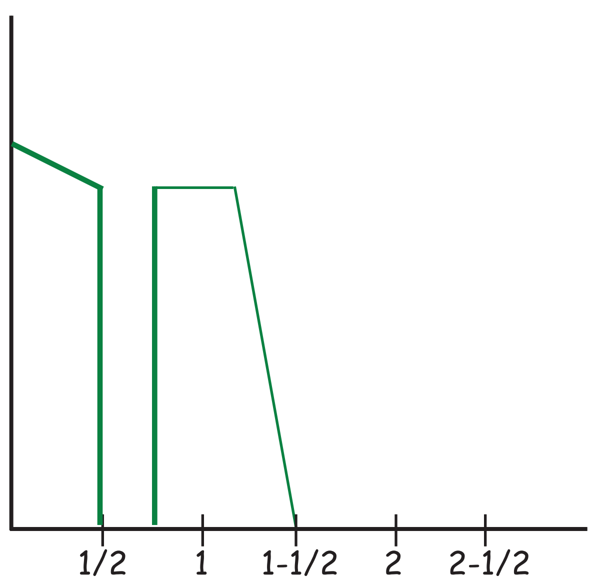


Graph 2



Graph 3

Make up a story
for this graph.



Instructions for the CBR

Walking with the CBR*

1. Turn on the calculator. Press **ON** (bottom left key).
2. Press the program key, **PRGM** (middle key in the third row).
3. EXEC and 1: Ranger are highlighted. Press **ENTER** (bottom right key).
4. prgm Ranger is on the screen. Press **ENTER.**
5. Texas Instruments is on the screen. Press **ENTER.**
6. Press **2** to Set Defaults.
7. Main Menu Start Now is on the top of the screen. Press **ENTER.**
8. Point CBR at target is on the top of the screen. Press **ENTER.**
9. Begin walking.
10. Study graph that appears on the calculator screen.
11. To repeat the process, press **ENTER.**
12. Press **3** to repeat the sample.
13. Press **5** to quit when you are finished,
14. To turn calculator off, press **2nd** (yellow key) and then press **ON.**

* The CBR, a Calculator Based Ranger, is used with a graphing calculator to collect, view, and analyze motion data.

Instructions for the CBR

Matching Graphs on the CBR*

1. Turn on the calculator. Press **ON** (bottom left key).
2. Press the program key, **PRGM** (middle key in the third row).
3. EXEC and 1: Ranger are highlighted. Press **ENTER** (bottom right key).
4. prgm Ranger is on the screen. Press **ENTER.**
5. Texas Instruments is on the screen. Press **ENTER.**
6. Press **3** for Applications.
7. Units is on the screen. Press **1** for meters.
8. Applications is on the screen. Press **1** for distance match.
9. Try to match . . . is on the screen. Press **1** for distance match.
10. Study the graph that is on the screen, so that your walk matches the graph.
11. Press **ENTER** when you are ready to walk.
12. You can now walk using the same graph or you can use a new graph. To do this press **ENTER**.
13. Options is on the screen. Press **1** for the same match, or **2** for a new match.
14. When you are done, press **QUIT** on the options screen.

* The CBR, a Calculator Based Ranger, is used with a graphing calculator to collect, view, and analyze motion data.

What Does It All Mean?

1. What does the horizontal-axis represent?______________________________
   What are the units of measurement?______________________________

2. What does the vertical-axis represent?______________________________
   What are the units of measurement?______________________________

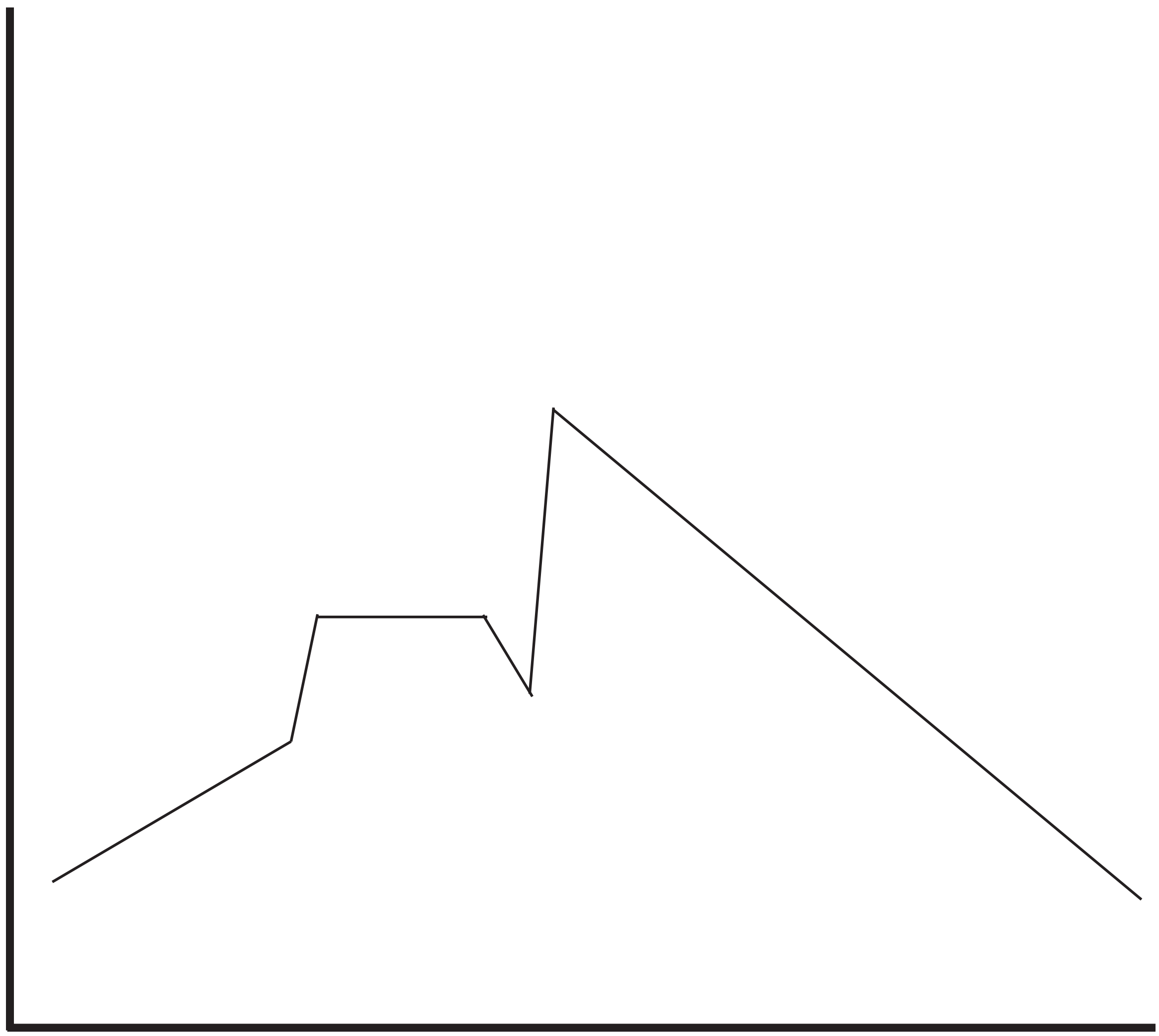
3. How do you decide where you should stand to begin?______________________________

4. What should a graph look like if you walk forward?__________Backward? __________

5. What should you do for a segment that is flat?______________________________

6. How do you decide how fast to walk?______________________________

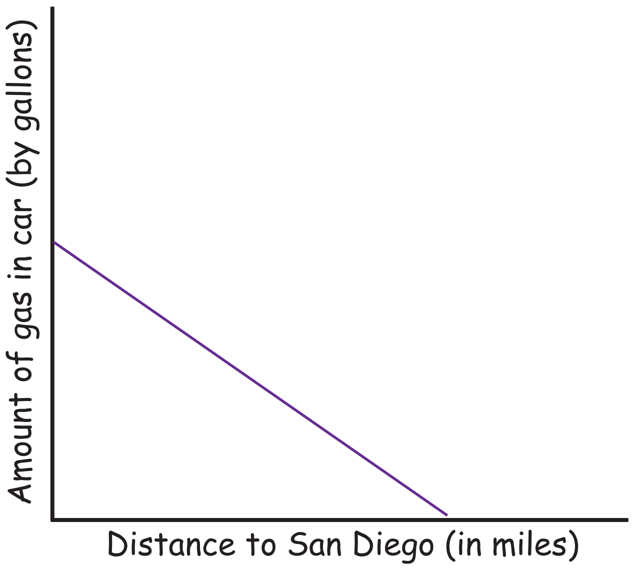
The Bathtub Graph



What Do They Tell You?

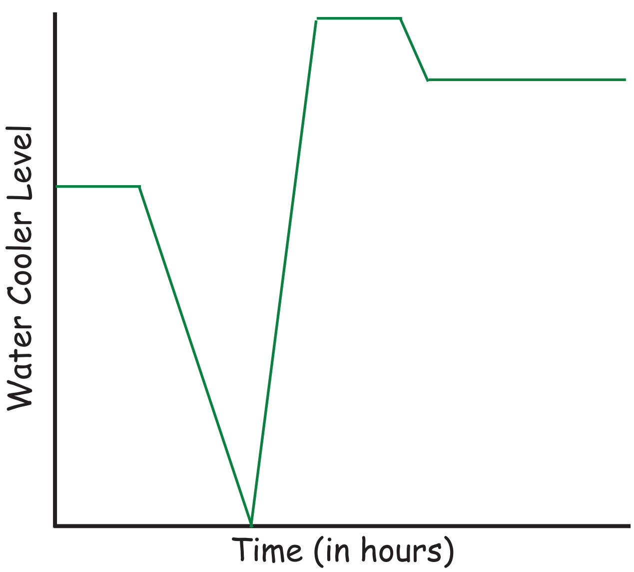
Each graph shows a relationship. Write a brief story for each graph.

1.

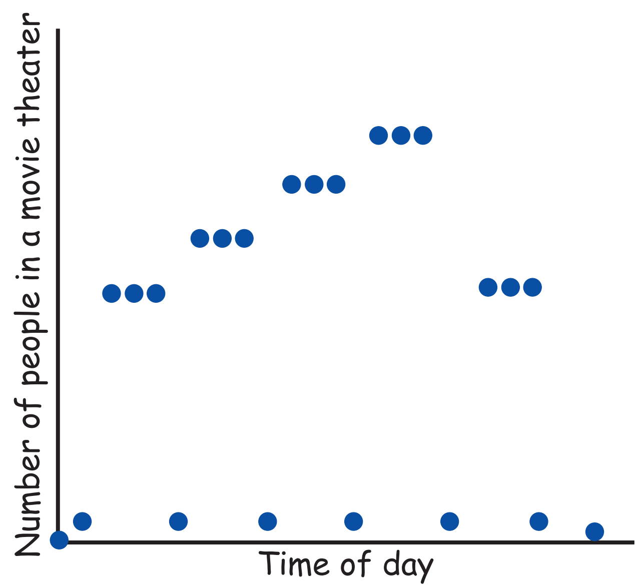


2.

3.



4.



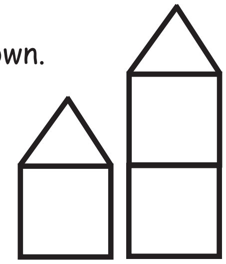
A Menu of Pattern Activities

1. Toothpick Houses

Materials: Flat toothpicks

A one and a two story toothpick house are built as shown.

How many stories can you build if you have 35 toothpicks?



Story Number	Number of Toothpicks
1	6
2	9
3	12
4	15
5	18

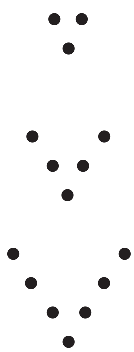
2. Flying Vs and Ws

Materials: Cubes, tiles, or similar objects

Birds and airplanes often fly in a shape that forms a **V** or **W**.

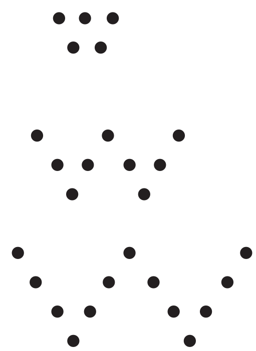
- Investigate the number of birds in the **V** shape and the number of airplanes in the **W** shape.
- Find the number of birds and the number of airplanes in the 10th pattern.

1.
2.
3.



Pattern Number	Number of Birds
1	3
2	5
3	7
4	9
5	11

1.
2.
3.



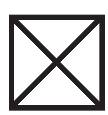
Pattern Number	Number of Airplanes
1	5
2	9
3	13
4	17
5	21

3. Diagonals of Polygons

Materials: Paper and pencil

Use what you know about patterns and relationships to find the number of diagonals in a decagon, a ten-sided figure.

Triangle =
3 sides
0 diagonals



Quadrilateral =
4 sides
2 diagonals

Sides	Diagonals
3	0
4	2
5	5
6	9
10	35

4. Cut-Cut

Materials: Paper and scissors

- Cut a sheet of paper in half and count the pieces.
- Cut each piece in half and count the pieces after the second cut.
- Continue to cut and count.

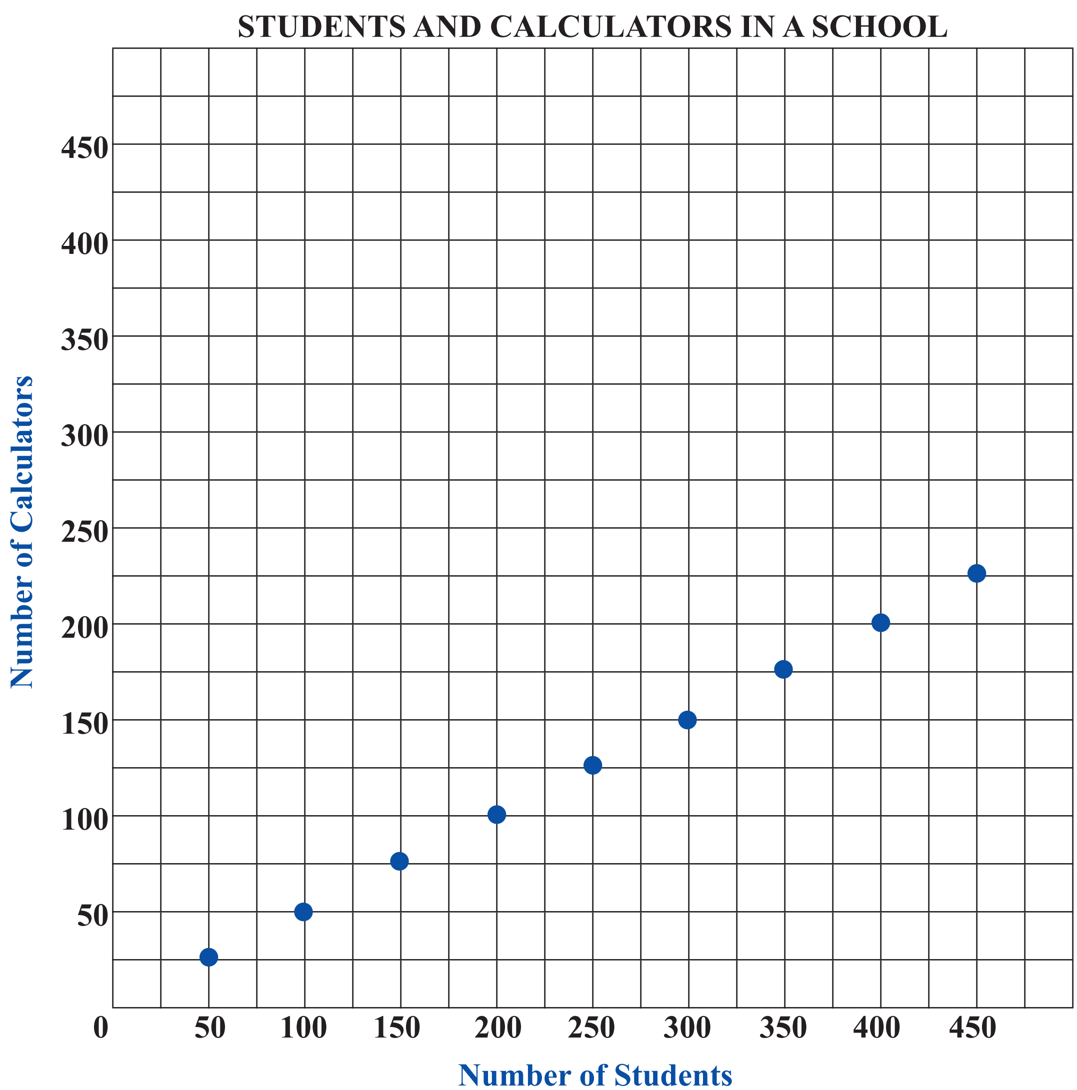
  How many pieces do you have after 5 cuts?

  How many would you have if you made 8 cuts?

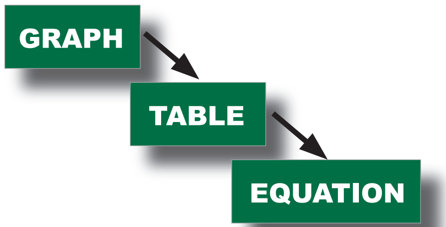
Number of Cuts	Number of Pieces
1	2
2	4
3	8
4	16
5	32

At Home with Graphs, Tables and Equations

1. Complete a table for the graph.
2. Write a rule or equation for the table.
3. Then, write a story for the graph.

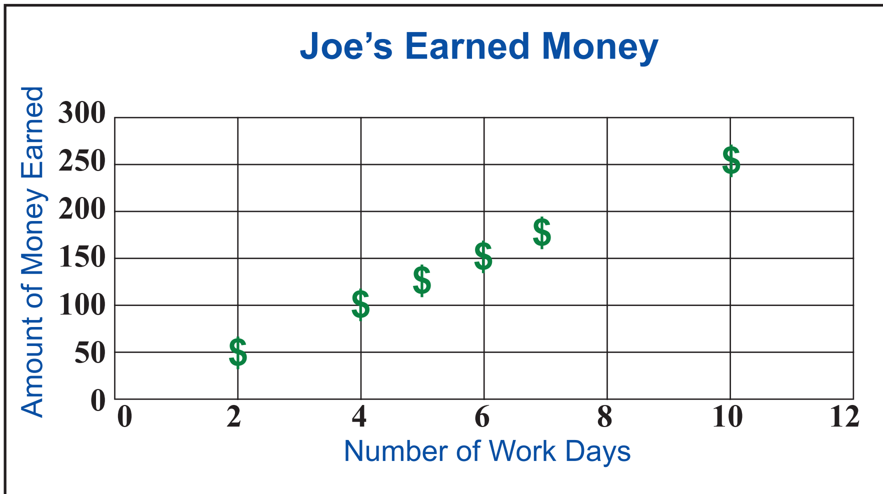


From Graphs to Tables to Equations

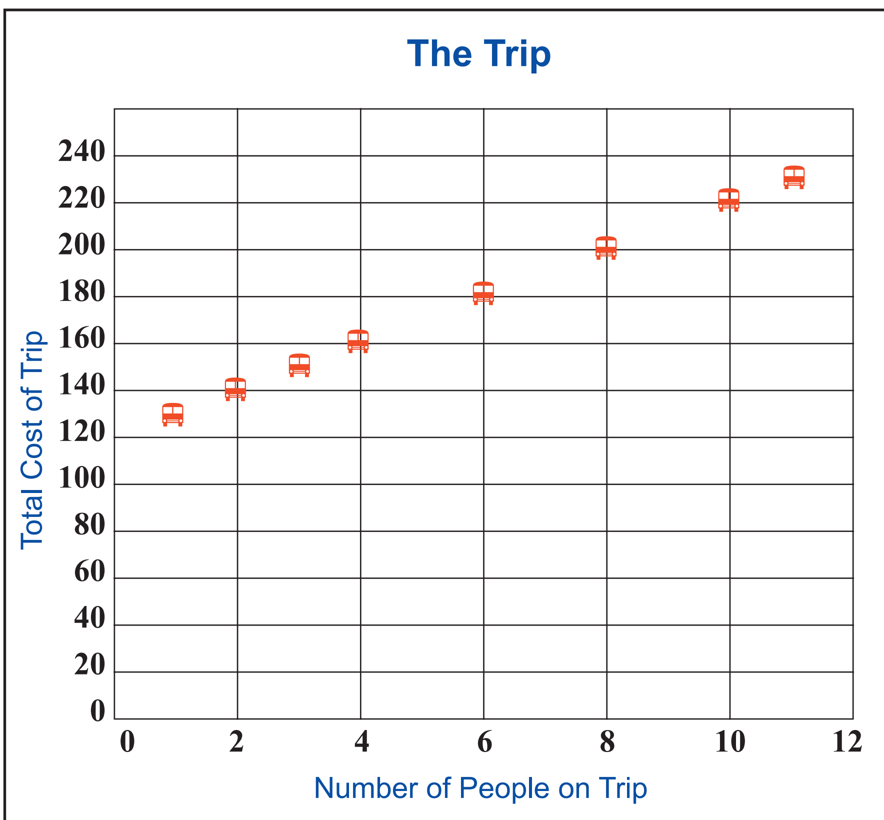


Use the graphs below to:

1. Complete the table
2. Find an equation for the table
3. Tell a story about each graph



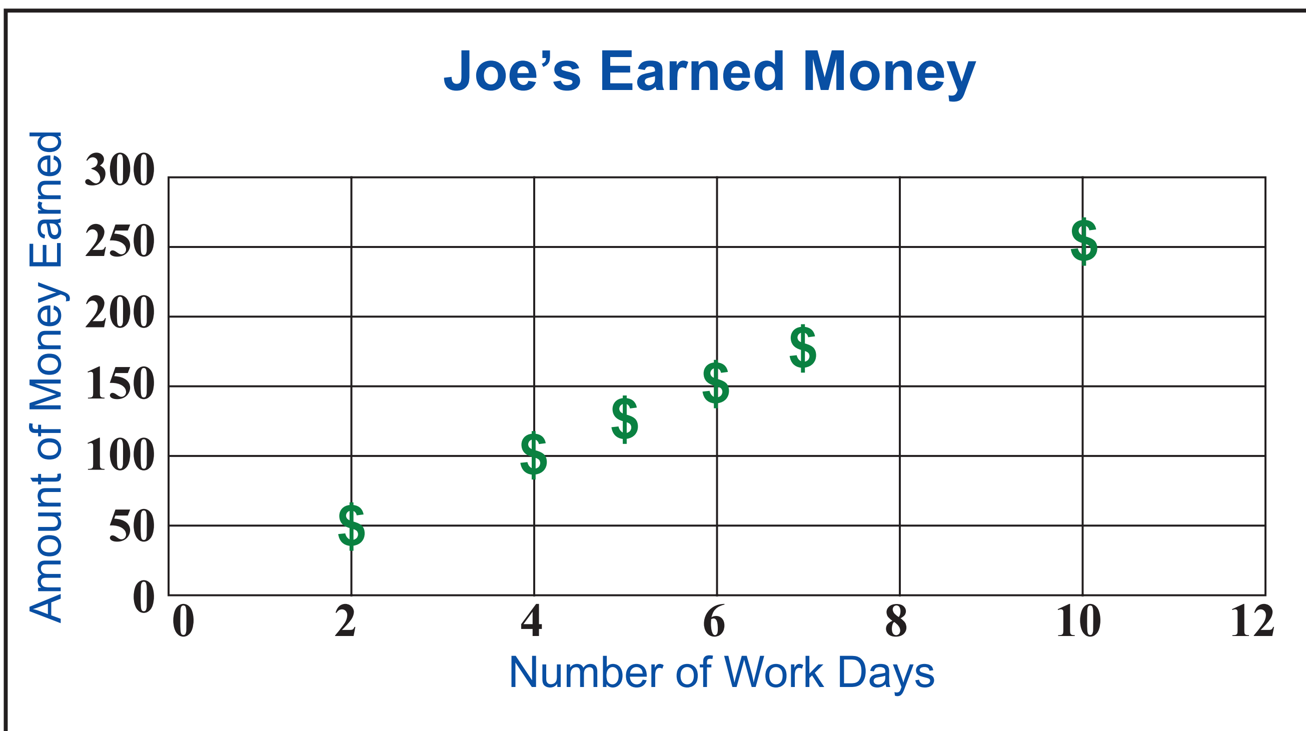
Number of Work Days	Amount of Money Earned



Number of People on Trip	Total Cost of Trip

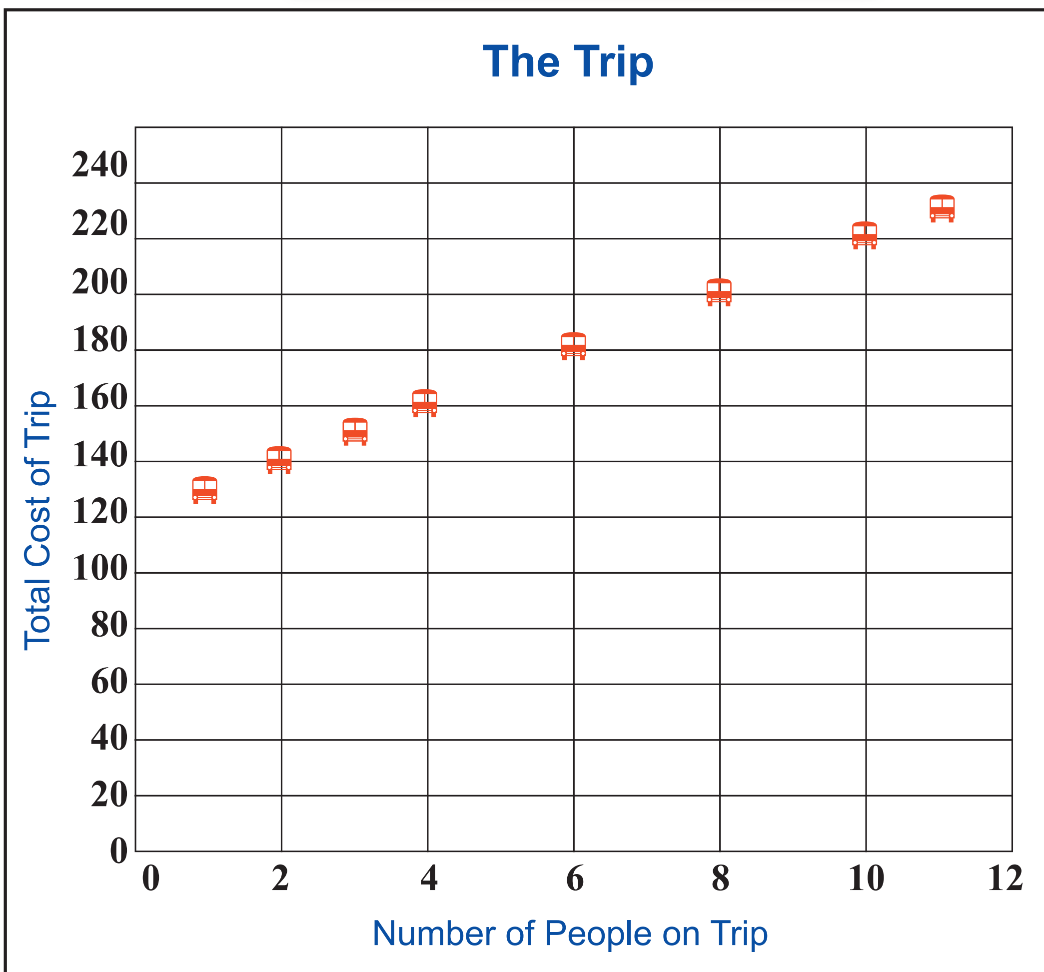
Joe's Earned Money

Number of Work Days	Amount of Money Earned

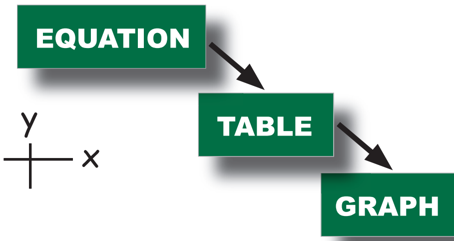


The Trip

Number of People on Trip	Total Cost of Trip



From Equations to Tables to Graphs



1. Complete a table for each equation. Graph your findings.

To complete the table:
- Choose a number for x or y
- Find the value of the other variable

To graph:
- Use graph paper
- Remember: y x

Table:

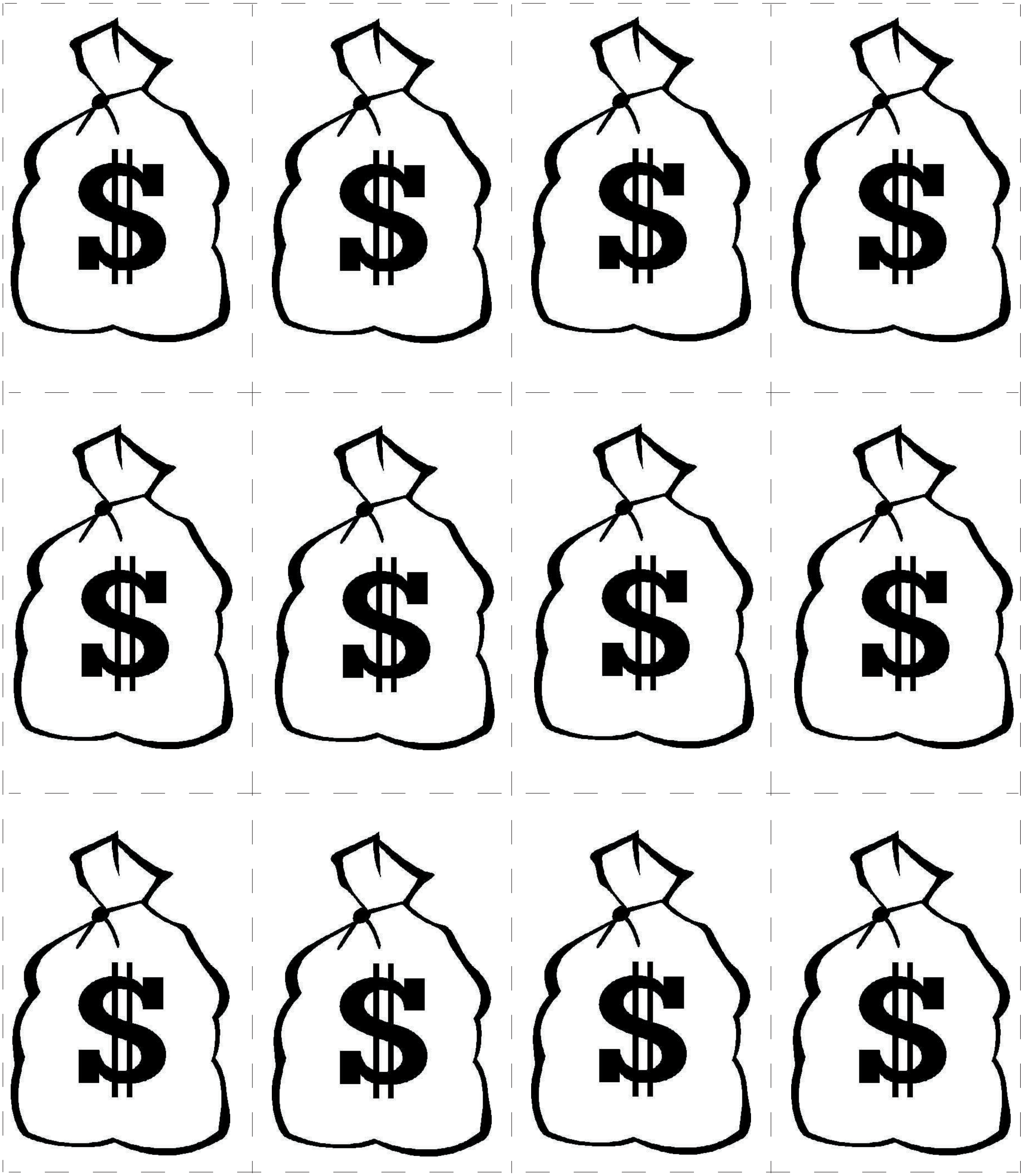
1a.	$x + y = 5$	2a.	$x = y$	3a.	$2x + 2y = 24$	4a.	$x^2 = y$
b.	$x - y = 2$	b.	$2x = y$	b.	$20 - 3x = y$	b.	$x^2 + 2 = y$
c.	$x + y = -1$	c.	$2x - 1 = y$	c.	$3x + 2 = y$	c.	$24 = x(y)$

2. Look for patterns in the graphs.
   How are the graphs alike? How are they different?

3. Some graphs slant to the right. Some slant to the left.
   Can you tell from the equation which ones will slant left? Right?

Mystery Bags

Cut out bags on dotted lines:



Bags of Gold

A pan balance has some combination of bags of gold and lead weights so that the two sides balance. Each bag has the same amount of gold. These bags of equal weight are called "mystery bags".

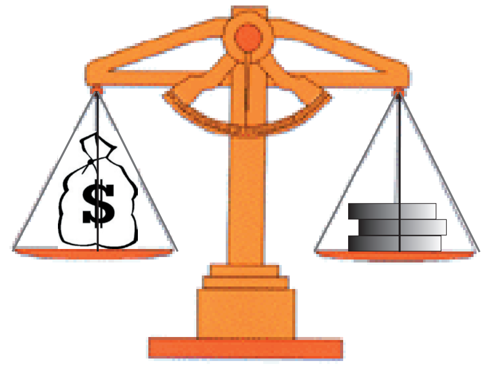


Figure out how much gold there is in each mystery bag. Explain how you know you are correct. You may want to use models or diagrams to show what you are doing.

1. There are 4 mystery bags on one side of the pan balance and 56 ounces of lead weights on the other side.
2. There is 1 mystery bag and 36 ounces of weights on one side, and 80 ounces of weights on the other side.
3. There are 10 mystery bags and 20 ounces of weights on one side and 100 ounces of weights on the other side.
4. There are 2 mystery bags and 18 ounces of weights on one side, and 3 mystery bags on the other side.
5. There are 8 mystery bags and 65 ounces of weights on one side and 3 mystery bags and 120 ounces of weights on the other side.
6. There are 5 mystery bags and 12 ounces of weights on one side and 5 mystery bags and 15 ounces of weights on the other side.
7. There are 12 mystery bags and 8 ounces of weights on both sides.

Adapted from *The Pit and the Pendulum*, Interactive Mathematics Program™, Key Curriculum Press

Familiar Algebraic Equations

As you solve these equations use your understanding from the bags of gold problems to explain your answer. You may want to refer back to your work on **A Menu of Pattern Activities** from session 1.

1. Remember the flying birds problem? You may want to go back to your notes on this.

   In the birds problem if ***p*** is the pattern number and ***b*** in the number of birds then the equation is 2p + 1 = b.

   - *If there are 89 birds, then the equation is 2p + 1 = 89. What is the pattern number?*
   - *If there are 55 birds, then the equation is 2p + 1 = 55. What is the pattern number?*
   - *If there are 30 birds, would there be a complete formation?*

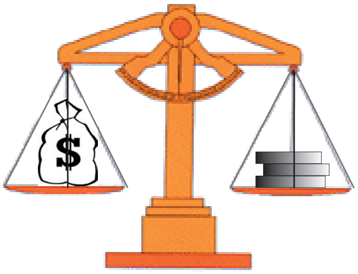
2. Then there was the Toothpick Houses problem. If ***s*** represents the number of stories and t represents the number of toothpicks, then the equation for this problem is 3s + 3 = ***t***.

   - *If there are 57 toothpicks, how many stories would the house be? Would you use all the toothpicks?*
   - *If there are 101 toothpicks, how many stories would the house have? Would you use all the toothpicks?*

3. Then there was the hexagon problem where you were looking for the perimeter of the joined hexagons. If ***h*** represents the number of hexagons and ***p*** represents the perimeter, what is the equation for this problem?

   - *Is it possible to have a perimeter of 30? 22? 106? Why or Why not?*

Solving Algebraic Equations



As you solve these equations use your understanding from the **Bags of Gold** problems to explain your answer. You may draw a picture, use manipulatives or solve symbolically.

1. $3x = 2x + 7$	2. $2m - 2 = 22$
3. $2y = -14$	4. $b + 3 = 2$
5. $5r + 44 = 9r + 2$	6. $36 = 3a + 6$
7. $108 + 11n = 7n + 4$	8. $12p + 21 = 10p + 26$
9. $-4 + 5g = -2 + 3g$	10. $4h + 7 = 23 + 4h$