SESSION SIX
EXPONENTIAL NOTATION

Outcomes

- To write the prime factorization of numbers in exponential form.
- To explore a comparison between exponential and linear growth.
- To engage in mathematical reasoning as it relates to exponential notation.

Overview

The sixth session of Thinking About Numbers reinforces prime factorization and introduces exponential notation. Repeated addition is again connected with multiplication; similarly repeated multiplication is connected with exponential notation. Participants explore applications for exponential growth and investigate the principles.

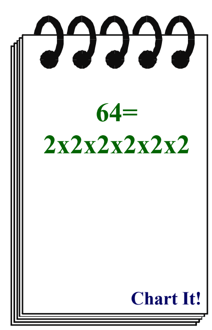
Time

10–15 minutes	The first part of the session allows participants to discuss their take home activities.
30–40 minutes	Participants look at multiplication as a shorthand method of writing repeated addition. They then look at prime factorization of 64 to see a need for a shorthand method to expression repeated multiplication. They have fun creating their own notation before the official notation is introduced. Participants practice exponential notation with prime factorization of several numbers.
30–40 minutes	Participants are given an application problem. They are to decide which amount is greater, \$1,000,000, \$5,000 a month, or money that doubles every day.
15-20 minutes	Participants investigate mathematical reasoning by exploring the principles of exponential notation.
15–20 minutes	In the closing activity, participants will reflect on what they found to be the most fun.

Materials

Facilitator	Transparencies (English & Spanish)
• Optional: overhead calculator	**No Transparencies**
Participant	**Handouts (English & Spanish)**
• Calculators	**Two per participant for class and home** *BLM 33: Prime Factorization Using Exponential Notation* *BLM 34: Applications of Growth* *BLM 35: Principles of Exponential Notation* **One per participant for home** *BLM 36.1-3: Bringing Mathematics Home 6* *BLM 37: Exponential Notation*

Activities

Preparation of Classroom	Notes
1. Set up the **Chart It!** 2. Place the name cards from last class near the front of the room where participants can easily find them.	
Discussion of Homework (10-15 minutes)	
1. Have participants discuss the following question: *What did you learn about strategies when doing The Mystery of Numbers with your children?* Take a few minutes for them to share it with their groups and then ask for a few volunteers to share with the whole class. 2. Answer any questions that the participants have about the **Prime and Composite Numbers** worksheet. • Take the time to talk about their discovery of patterns of primes that they may have encountered when finding the primes from 1 – 300. • Discuss their discovery when evaluating 6n – 1.	There are no specific icebreakers in this and the remaining sessions. If your parents are not relaxed and laughing together, you need to continue building the community. Open with icebreakers that will help them to get to know each other better. If it does not come up talk about how prime numbers are odd except for 2. Ask them why that is? Did any of their prime numbers end in 5? Why or why not?
Exponential Notation (30-40 minutes)	
Introduction to exponents 1. Put up on the board 2 + 2 + 2 + 2 + 2 + 2. Ask: *Is there a short cut way to write repeated addition?* Participants will remember that multiplication is a short-hand way to write repeated addition. For instance, 2 + 2 + 2 + 2 + 2 + 2 can be written as 6 x 2. 2. Say: *Mathematicians are always looking for shorter ways to write repeated operations. Over the next few sessions we will learn some of the short-hand methods for repeated operations. Lets look at another operation.* 3. Repeated Multiplication • Have participants find the prime factor of 64. • Give enough time for participants to check within their group and make sure everyone feels comfortable using the factor tree. • Have a participant share their answer on the overhead showing the factor tree. (Important: Pick someone who has not used exponents to express their answer).	 After the activity add exponential notation: $64 = 2^6$

Activities

Exponential Notation (continued)

- Tell them that this is a lot of two's.
- Write the number 4096 on the overhead and write the factorization.
- Hopefully at this point, they will notice that a short cut would be helpful.
- Tell them that we are going back in history. We are in Greece at approximately 300 BC. We have a dilemma. We are exploring prime factorization and we need a short-hand method. Archimedes one of the great mathematicians is sitting with us (we are also famous mathematicians - how much fun would this be if you brought in sheets for people to dress in togas?). We need to choose a method that will be used throughout the rest of time.
- In your groups invent a shorter way to write this repeated multiplication for the number 64. (If a group knows exponential notation encourage groups to come up with a way other than exponents.)
- Have groups display their short cuts on chart paper (1/4 sheet each group). Encourage creativity.
- The richness in this activity will come out of the imaginative ways that groups have come up with. This should be a fun activity.
- Enjoy the variety of the presentations.
- They discuss the pros and cons of each method.
- Ask:
  Who can tell us what Archimedes and his group chose?
- At this time confirm that exponential notation is the correct short-hand to show repeated multiplication.

Guided practice writing exponential notation

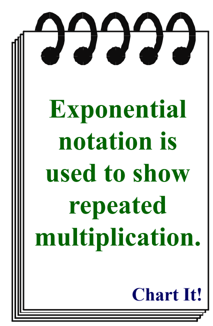
1. Hand out **Prime Factorization using Exponential Notation**

2. Question 1. (4, 8, 125, 243)
   - Have participants do question 1
   - Share answers

3. Introduce multiple bases.
   - Ask participants how they would write 2 x 2 x 3 x 3 using exponential notation.
   - Have them try 2 x 2 x 2 x 3 x 3 x 23 x 23
   - Have participants continue with the practice in question 2.

Notes

Optional: Have participants find the prime factorization of 4,096.
4096 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2.



Activities

Exponential Notation (continued)	Notes
• Have everyone share their answers. 4. Partners share • Get with a partner • Have partner 1 tell partner 2: What is the difference between 4 x 2 and 4^2? • Then have partner 2 tell partner 1. • Then do this for 4 x 3 and 4^3] • Ask them to think about what power of 4 would be larger than 1000.	If one partner struggles with this concept more, it is sometimes helpful for him/her to go first. The time to talk it out will sometimes solidify the concept.
Applications of Growth (30-40 minutes)	
1. Say: *We have now seen some of the exponential growth. What does it all mean? Lets look at a situation like Aunt Bessie (75 years old) is drawing up her will and wants to put you in it. She told me that this is a secret that must be kept quiet. She said "My dear, you may choose which you would rather have: one million dollars today, five thousand dollars every month for the rest of my (Aunt Bessie's) life, or I will give you 2 cents today, 4 cents tomorrow, 8 cents the next day and keep doubling the new amount for a total of 28 days." Without doing any mathematics which would you choose?* 2. Take a tally of votes for the three options and write on chart paper 3. Since calculators are needed to solve the following problem, it is important to take the time to show the participants how to do repeated multiplication and addition on the calculator. • Instruct them to multiply by 2 repetitively by entering 2 x 2 = x 2 = = = • Ask them how they would add 5000 repetitively in the same manner. 4. Now let them work this problem in their groups. Have them consider all three options explaining the benefits of each. Then have them choose the option they would take. 5. Have them prepare a presentation on chart paper. 6. Ask: *When does this have to do with prime factorization and exponential notation?*	= represents the enter button as well Aunt Bessie's proposal: $1,000,000: Things to consider might be the tax for this gift since Aunt Bessie is still alive, interest that it will accumulate during her life time. $5,000 a month: Things to consider might be Aunt Bessie's life span and considering that of the $60,000 only $10,000 is tax free as a gift. Also they might consider investment returns. 2 cents doubled: • 2^{28} = 268,435,456 however that is in pennies so in dollars one would collect $2,684,354.56 • Remember that Aunt Bessie is handing you the money each day and so you also have the money accumulated each day. This amounts to $2,684,354.54. When added to $2,684,354.56 the grand total is $5,368,709.10 (this amounts to 2 cents less than 2 x 2). • If participants do not bring up the accumulation of money you might consider skipping that part of the problem.

Activities

Principles of Exponential Notation (10 minutes)	Notes
Hand out **Principles of Exponential Notation.** • Give the participants time to read the questions to see if they understand them. • Answer any questions that come up. • Give participants time to complete the two problems. • Have different groups share their reasoning for each of the problems.	If time permits play the game that is in **Bringing Mathematics Home 6.**
Closure (10 minutes)	
Participants reflect on the session. Ask: *What did you find to be the most fun today?* Ask a few volunteers to share their reflections	
Take Home Activities (5 minutes)	
1. There are five handouts for participants to take home: • **Bringing Mathematics Home 6** • **Exponential Notation** • **Prime Factorization Using Exponential Notation** • **Applications of Growth** • **Principles of Exponential Notation.** 2. Have participants look through the packet of materials as you explain them. The object of the take home activities is for them to practice with their children. Therefore, they need fresh copies of the session's activities. 3. Let participants know that they should be ready to share their experiences at the next session. 4. If you have time play the game.	Secret problem: 100 people = 6 days 1000 people = 9 days The world: population is about 6 billion so it will take about 32 days.
Preparation for the Next Session	
1. Collect name cards for use in the next sessions. 2. Save the **Chart It!** and bring it to the next class. If desired, you may have the log typed and distributed to participants at the next class.	