

## SESSION FOUR

### DIVISORS (FACTORS) AND MULTIPLES

#### Outcomes

- To use pennies and Cuisenaire® rods to define and explore divisors and multiples
- To experience application problems that require divisors and multiples
- To engage in mathematical reasoning as it relates to multiples and divisors

#### Overview

The fourth session of Thinking About Numbers focuses on divisors and multiples. A deeper understanding of the concepts is developed through the use of area models, manipulatives, and applications. As in other sessions, there is a section on investigations of mathematical principles.

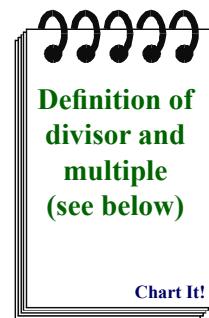
#### Time

- 10-15 minutes** The first part of the session allows participants to discuss their take home activities.
- 30-40 minutes** Participants are engaged in a story problem that sets the stage for the concepts of the session. By doing this activity, the terms divisor and multiple are introduced. Further explorations include creating trains with Cuisenaire® rods and arranging pennies.
- 30-40 minutes** To develop the concept of divisors, participants investigate with pennies and area models. Greatest common factors are discovered through an area model and through recording dimensions of rectangles.
- 15-20 minutes** Participants investigate mathematical reasoning by exploring true/false statements
- 15-20 minutes** In the closing activity, participants are taught games that support the concepts of the session.

#### Materials

Facilitator	Transparencies (English & Spanish)
<ul style="list-style-type: none"> <li>• A set of Cuisenaire® rods for the overhead</li> </ul>	<i>BLM 21: 24 by 36 Grid</i>
Participant	Handouts (English & Spanish)
<ul style="list-style-type: none"> <li>• Pennies</li> <li>• Cuisenaire® rods</li> <li>• Graph paper (eight copies of <b>BLM 21: 24 by 36 Grid</b>)</li> </ul>	<p><b>Eight per participant for class</b>  <i>BLM 21: 24 by 36 Grid</i></p> <p><b>Two per participant for class and home</b>  <i>BLM 22: Concepts of Multiples</i>  <i>BLM 23: Concepts of Divisors</i>  <i>BLM 24: Principles of Divisors and Multiples</i></p> <p><b>One per participant for home</b>  <i>BLM 25: Bringing Mathematics Home 4</i>  <i>BLM 26: The Factor Game</i>  <i>BLM 27: Divisors and Multiples of Whole Numbers</i></p>

## Activities

Preparation of Classroom	Notes
<ol style="list-style-type: none"> <li>1. Since there are several take home activities in this session, it would help to make a packet for participants ahead of time.</li> <li>2. Set up the <b>Chart It!</b></li> <li>3. Place the name cards from last class near the front of the room where participants can easily find them.</li> <li>4. Have a supply of Cuisenaire® rods, graph paper, and pennies on the tables.</li> </ol>	
Discussion of Homework (10-15 minutes)	
<ol style="list-style-type: none"> <li>1. Have participants discuss the following question: <i>What is one thing that you learned about your child through doing the Bringing Mathematics Home-3 work?</i></li> </ol> <p>Take a few minutes for them to share it with their groups and then ask for a few volunteers to share with the whole class.</p> <ol style="list-style-type: none"> <li>2. Answer any questions that the participants have about <b>Division with Whole Numbers</b>.</li> </ol>	<p>There are no specific icebreakers in this and the remaining sessions. If your parents are not relaxed and laughing together, you need to continue building the community. Open with ice breakers that will help them to get to know each other better.</p>
The Concept of Multiples (30-40 minutes)	
<p><b>Introduction</b></p> <ol style="list-style-type: none"> <li>1. Start this section off with a story similar to: <i>This Fourth of July I am serving hot dogs. I noticed when I went shopping that hot dogs come 10 to a package and buns come 8 to a package. I don't want any hot dogs without buns. I want to get enough hot dogs and hot dog buns so they match exactly so that every hot dog has a bun and every bun has hot dog. What is the smallest number of packages of each that I need to buy? How many hot dogs with buns do I have?</i></li> </ol> <ul style="list-style-type: none"> <li>• Have participants work on the problem in their groups.</li> <li>• Share their solutions</li> <li>• Ask: <i>What do, 8 and 10 have in common in this problem?</i></li> <li>• Use the responses from the participants to generate the discussion that these are all divisors of 40. Define divisors as numbers that can be divided evenly into another number with no remainder. Chart It!</li> </ul>	<p>The solution can look like:  <b>10:</b> 10, 20, 30, 40 (4 packages)  <b>8:</b> 8, 16, 24, 32, 40 (5 packages)</p> <div data-bbox="1045 1262 1252 1577">  <p><b>Definition of divisor and multiple (see below)</b></p> <p><b>Chart It!</b></p> </div> <p>If 8 is divisible by 2, then 2 is called a divisor, and 8 is called a multiple. 2 divides 8 evenly and so is a divisor, and 8 is <math>2 + 2 + 2 + 2</math>: in other words, multiple 2's.</p>

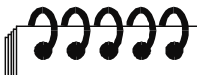
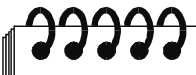
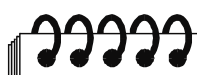
## Activities

The Concept of Multiples (continued)	Notes
<ul style="list-style-type: none"> <li>Also, state that 40 is a multiple of 8 and 10. Since 10 divides into 40 evenly, 40 is a multiple of 10. Let's see why. Let's count by 10's. When we do that we are naming the multiples of 10 and we get to 40 so 40 is one of them.</li> </ul> <p><i>We know that 40 is a multiple of 8 and 10. See if you can find any other numbers of which 40 is a multiple.</i></p> <p>2. Tell the participants:</p> <p><i>During this session we are going to examine this situation mathematically. In order to do that we will be studying divisors and multiples. We will use story problems, area models and manipulatives to help us understand these concepts.</i></p> <p><b>Multiples with Manipulatives</b></p> <p>1. Hand out <b>Concepts of Multiples</b>.</p> <p>2. Demonstrate how to find the answer to the hot dog question using Cuisenaire® rods. Instruct the participants to lay the different colored rods out according to length. Using the smallest rod, figure and record the length of each color. Now have them find the 8 and 10 unit rods. Have them form a train using the 8 unit rods and another train using the 10 unit rods. Build the two trains until their lengths coincide.</p> <p>Ask:</p> <ul style="list-style-type: none"> <li><i>How many rods did it take?</i></li> <li><i>How does this relate to our hot dog question?</i></li> <li><i>Predict when they will next coincide. Check your prediction by building it.</i></li> <li><i>Name some other amounts where the hot dogs and buns would match.</i></li> <li><i>Out of the numbers that we have been using, name as many multiples and divisors as you can. Record each, stating the two numbers that form the relationship. Example: 5 is a divisor of 10. 10 is a multiple of 5.</i></li> </ul> <p>3. Have the participants look at problem 2, which reads: Margie and Bernadette want to buy a stereo for their room. They have decided that they will work after school. Margie earns \$7 an hour and Bernadette earns \$4 an hour. Margie wants to work only long enough to match Bernadette.</p>	<p>Here are the lengths of the Cuisenaire® rods:</p> <ul style="list-style-type: none"> <li>1: tan</li> <li>2: red</li> <li>3: light green</li> <li>4: purple</li> <li>5: yellow</li> <li>6: dark green</li> <li>7: black</li> <li>8: brown</li> <li>9: blue</li> <li>10: orange</li> </ul>

## Activities

The Concept of Multiples (continued)	Notes
<ul style="list-style-type: none"> <li>How long will each girl need to work so that they each contribute the same amount?</li> <li>What is that amount?</li> </ul> <p>Have them work with a partner to solve this problem. They need to be ready to defend their answer.</p> <p>4. Have the participants work on #3 and #4 on their own. Ask for volunteers to present their solutions and their strategies.</p>	<p>In problem #4: The product of these numbers is a natural choice for the answer. Since <math>2 \times 3 \times 4 \times 5 = 120</math> which is a possible answer. The students may very well think they are finished. It is important to point out that there may be another answer. The only way to get the other possible answer, 180, is to recognize that 60 is also a common multiple of 2, 3, 4, and 5. While this is somewhat sophisticated it is definitely within the reach of the students who have gone through this lesson carefully. This kind of exploratory thinking is important in mathematics.</p>
The Concept of Divisors (30-40 minutes)	
<ol style="list-style-type: none"> <li>Have participants build as many arrays as possible with 24 pennies. Have them record the dimensions of each array that they build.             <ul style="list-style-type: none"> <li>Have participants share their results</li> <li>Chart the dimensions</li> </ul> </li> <li>Have participants build as many rectangles as possible with 36 pennies. Have them record the dimensions of each array that they build.             <ul style="list-style-type: none"> <li>Have participants share their results</li> <li>Chart the dimensions</li> </ul> </li> <li>Quilting Activity             <ul style="list-style-type: none"> <li>Display 24 by 36 grid on the overhead.</li> <li>Illustrate making 2 by 2 squares that fit the grid. Show how they cover the entire grid and that they are all the same size.</li> <li>Hand out about 8 grids per person and have them explore to see what other sized squares will fit into the grid.</li> <li>Have the participants record the squares that they have found.</li> <li>Have volunteers share the squares that were found.</li> </ul> </li> <li>Connecting the activities:             <ul style="list-style-type: none"> <li>Have participants go back to question #1 and #2.</li> <li>Have them circle the lengths and widths that 36 and 24 have in common.</li> </ul> </li> </ol>	<p><b>24:</b></p> <p>1 by 24 2 by 12 3 by 8 4 by 6</p> <p><b>36:</b></p> <p>1 by 36 2 by 28 3 by 12 4 by 9 6 by 6</p> <p>The participants should find that 24 and 36 have 1, 2, 3, 4, 6, and 12 in common. These should also be the dimensions of the squares that they found in #3.</p>

**Activities**

The Concept of Divisors (continued)	Notes
<ul style="list-style-type: none"> <li>Have them describe the relationship between these numbers and the size of the squares that they found in question #3.</li> <li>Congratulate them on finding the common divisors of 24 and 36.</li> <li>Ask participants which divisor is the greatest common divisor, and how they know.</li> </ul> <p>5. Alternative way to check the Greatest Common Divisor</p> <ul style="list-style-type: none"> <li>Start with a 10 by 15 grid.</li> <li>Cut the largest perfect square that is possible off of the end of the grid. It should be a 10 by 10 because it should be the same height as the grid.</li> <li>You are left with a 5 by 10 grid. Again, cut the largest square off one end of the grid (This time it will be a 5 by 5).</li> <li>The remaining 5 by 5 square is the dimension of the greatest common divisor: 5.</li> <li>Have participants check their answer to #5 with a 24 by 36 grid.</li> </ul> <p>6. Have participants work on #6 with partners. Have them prepare a chart for one of the problems and present it.</p> <p>7. Have the participants turn to their partners and explain the difference between divisors and multiples as if their partner had never heard the words before.</p> <p>8. Explain to the participants that divisors are more commonly called factors.</p>	<div data-bbox="1031 262 1242 577">  <p><b>Definition of common divisor</b></p> <p>Chart It!</p> </div> <div data-bbox="1282 262 1494 577">  <p><b>Greatest Common Divisor</b></p> <p>Chart It!</p> </div> <div data-bbox="1153 976 1364 1291">  <p><b>Divisors are called factors</b></p> <p>Chart It!</p> </div>
Principles of Divisors and Multiples (15-20 minutes)	
<p>Hand out <b>Principles of Divisors and Multiples</b>.</p> <ul style="list-style-type: none"> <li>Give the participants time to read the questions to see if they understand them.</li> <li>Answer any questions that come up</li> <li>Give participants time to complete the three problems.</li> <li>Have different groups share their reasoning for each of the problems.</li> </ul>	

## Activities

Closure (10 minutes)	Notes
<p>Participants reflect on the session. Ask:</p> <p><i>What about today's activities have been enlightening to you about divisors and multiples?</i></p> <p>Ask a few volunteers to share their reflections</p>	
Take Home Activities (5 minutes)	
<ol style="list-style-type: none"> <li>There are six handouts for participants to take home:             <ul style="list-style-type: none"> <li><b>Bringing Mathematics Home 4</b></li> <li><b>The Factor Game</b></li> <li><b>Divisors and Multiples with Whole Numbers,</b></li> <li><b>Concepts of Divisors</b></li> <li><b>Concepts of Multiples</b></li> <li><b>Principles of Divisors and Multiples</b></li> </ul> </li> <li>Have participants look through the packet of materials as you explain them. The object of the take home activities is for them to practice with their children. Therefore, they need fresh copies of the session's activities.</li> <li>If time permits, play the two games so that participants will be familiar with them.</li> <li>Let participants know that they should be ready to share their experiences at the next session.</li> </ol>	
Preparation for the Next Session (5 minutes)	
<ol style="list-style-type: none"> <li>Collect name cards for use in the next sessions.</li> <li>Save the <b>Chart It!</b> and bring it to the next class. If desired, you may have the log typed and distributed to participants at the next class.</li> </ol>	