

SESSION THREE WRITING EQUATIONS

Outcomes

- To use strategies to record patterns and relationships
- To use models and tables to write an algebraic equation for a pattern
- To become familiar with and apply various problem solving strategies

Overview

This session reviews writing expressions and introduces writing equations. This is at the discretion of the instructor. Time is also spent on problem solving and sharing strategies.

Time

- | | |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5-10 minutes | The session begins with participants using words to complete statements that require multiplication or division. Participants share their work from the take home activities. |
| 15-20 minutes | The words in Writing Equations are translated to algebraic expressions and equations. Participants present their work on this. |
| 20-30 minutes | Participants are given problems to solve and asked to write an expression (equation) for each. The situations model linear relationships. Participants present their work. |
| 10-15 minutes | Discussion follows on the various strategies used to solve the problems. A list of strategies is generated by the participants. This is compared to a handout of problem solving strategies. There is discussion on how their children might use these strategies when doing mathematics. Participants use strategies to solve some non-linear relationship problems. |
| 5-10 minutes | In the closing activity, parents reflect on what insights they had during the session about equations or problem solving. Take home activities are assigned. |

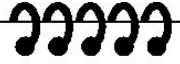
Materials

Facilitator	Transparencies (Eng. & Spanish)
<ul style="list-style-type: none"> • Poster of Writing Equations on chart paper (optional) • Overhead manipulatives for the Money Exchange Game (e.g. pennies and nickels, two-color counters, two different color tiles, etc.) 	<p><i>BLM 9: Writing Equations</i> <i>BLM 11: Problem Solving Strategies</i> <i>BLM 14: Money Exchange Game</i></p>
Participant	Handouts (English & Spanish)
<ul style="list-style-type: none"> • Pattern Blocks, 20-30 per table (red trapezoids will be used) • Colored Cubes (any type), 20-30 per table • Colored Tiles, 20-30 per table • Pennies and nickels (provided by participants) 	<p>One per participant for class <i>BLM 9: Writing Equations</i> <i>BLM 10.1-2: Equations and Problem Solving</i> <i>BLM 11: Problem Solving Strategies</i> <i>BLM 12: Bags of Cookies</i></p> <p>One per participant for home <i>BLM 13: Engaging Your Children in Problem Solving</i> <i>BLM 14: Money Exchange Game</i></p>

Activities

Preparation of Classroom	Notes
<ol style="list-style-type: none"> 1. Post charts from previous sessions to use as needed throughout the session. 2. Have copies of Writing Equations on the table. Have written instructions to begin this activity displayed on the overhead projector. 3. Set up the Chart It! 4. Distribute name cards (if applicable). 	<p>If it is a bilingual setting, be sure to include both languages on the chart. Alternate languages so that the same language does not always appear on top.</p>
Discussion of Homework (5-10 minutes)	
<ol style="list-style-type: none"> 1. As participants arrive have them complete the statements on the Writing Equations handout. It is not necessary for each participant to complete all the statements before discussion begins. 2. As they work, take the time to talk to each participant individually about their previous assignment. This is a good time to: <ol style="list-style-type: none"> a) Determine who completed the problems. b) Determine the questions that they have. c) Choose a few participants to share their work on the homework problems. Hand them a transparency to record their work and the algebraic expression for one of the problems. 3. Have participants present their problem. 4. Important questions to ask are: <ul style="list-style-type: none"> • <i>Did anyone write a different expression?</i> • <i>How is this expression different from the other one?</i> • <i>Where did this ___ come from?</i> 	<p>Even though this activity can be completed without giving copies to the participants, the copies enable participants to ask questions individually if needed, and have a reference for later use.</p> <p>Some expressions for Toothpick Houses:</p> <ul style="list-style-type: none"> • $3 + 3x$ the roof plus 3 for every floor • $6 + 3(x - 1)$ the top floor with the roof plus 3 for every other floor • $3 + 4x - x$ the roof plus 4 for every floor minus the 1 that has already been counted <p>It is not necessary to have more than one presentation of a problem unless different expressions or strategies have been used to solve it.</p>
Completing Statements / Using Variables (15-10 minutes)	
<ol style="list-style-type: none"> 1. Tell participants that last session they worked at writing expressions for problems like the ones on Writing Equations. This session, they will be going a step further by writing equations for these problems. 2. Model writing an algebraic expression for the example problem by saying: <ul style="list-style-type: none"> • <i>If we had 6 hours, what would we need to do to find the number of minutes? 10 hours?</i> • <i>What variable can we use to represent the number of hours?</i> • <i>So, for any number of hours, what would we need to do to find the number of minutes?</i> 	<p>The purpose of Writing Equations is to review the use of variables. It is also be used to introduce the concept of an equation. Based on the class' needs and experiences the facilitator can decide not to introduce equations at this time, doing further work with expressions instead.</p>

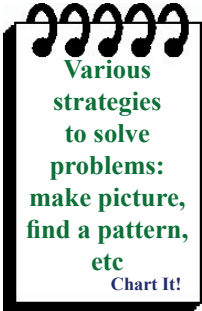
Activities

Completing Statements / Using Variables (continued)	Notes																
<ul style="list-style-type: none"> • How can we write an expression that shows how to find the number of minutes? • Does anyone have a different way of writing this? ($h \times 60$) <p>Record this on the overhead transparency of Writing Equations. Remind them that in the last session they wrote algebraic expressions with multiplication in various ways. Refer to the Chart It! page from the last session.</p> <p>3. Introduce the concept of an equation by saying: <i>We have been working with expressions. We now want to use expressions to write equations. The difference between an expression and an equation is like the difference between a phrase and a sentence. To turn an expression into an equation, you have to tell what you get when you find the value of the expression. So let's look at our expression $60 \times$ hours.</i></p> <p>Now ask: <i>What is it that we get when we take $60 \times h$? ($h \times 60 = ?$)</i></p> <p>4. Write $h \times 60 = \#$ of minutes on the transparency, circle # of minutes. and say: <i>If $h \times 60 = \#$ of minutes, then to shorten this sentence assign # of minutes a variable. What variable do we want to use?</i></p> <p>5. Record $m = h \cdot 60$ on the overhead transparency, Writing Equations. Say: <i>So the number of minutes, m, is equal to the number of hours, h, times 60.</i></p> <p>6. Let them know that $m = h \times 60$ is called an algebraic equation. Record the term and write the equation next to the term.</p> <p>7. Comment on the difference between expressions and equations. Record $h \times 60$ and write algebraic expression next to it.</p> <p>8. Have the participants work in pairs to write algebraic equations for each of the problem in Writing Equations. Let the participants know that they have about 5 minutes to do this.</p> <p>9. Ask for volunteers to record their equations for each problem on the overhead transparency. Have participants who have other ways to write the same equation share their work.</p>	<p>Some of the ways to write multiplication that have been discussed are:</p> <table style="width: 100%; border: none;"> <tr> <td style="padding-right: 20px;">$h \times 60$</td> <td style="padding-right: 20px;">$60 \times h$</td> <td>$60h$</td> </tr> <tr> <td style="padding-right: 20px;">$h * 60$</td> <td style="padding-right: 20px;">$(60)(h)$</td> <td>$60 (h)$</td> </tr> </table> <div style="border: 2px solid black; padding: 10px; margin: 20px 0; text-align: center;">  <p style="color: green; font-weight: bold;">algebraic equation: $m = h \times 60$ algebraic expression: $h \times 60$</p> <p style="font-size: small; color: green;">Chart It!</p> </div> <p>Examples of Expression and Equations</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">Expressions:</td> <td style="width: 50%;">Equations:</td> </tr> <tr> <td>1. $24 \cdot d$</td> <td>1. $h = 24d$</td> </tr> <tr> <td>2. $25 (q)$</td> <td>2. $q \times 25 = p$</td> </tr> <tr> <td>3. $d / 7$</td> <td>3. $w = d/7$</td> </tr> <tr> <td>4. $m / 12$</td> <td>4. $m/12 = y$</td> </tr> </table> <p>This is a good time to discuss various methods of writing division problems.</p>	$h \times 60$	$60 \times h$	$60h$	$h * 60$	$(60)(h)$	$60 (h)$	Expressions:	Equations:	1. $24 \cdot d$	1. $h = 24d$	2. $25 (q)$	2. $q \times 25 = p$	3. $d / 7$	3. $w = d/7$	4. $m / 12$	4. $m/12 = y$
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Activities

Exploring Equations (40 - 50 minutes)	
<ol style="list-style-type: none"> 1. Distribute copies of Equations and Problem Solving. Read each problem aloud. Make sure that each problem is understood before the participants begin their work. 2. Ask participants to work in pairs or small groups. Have them choose any of the three problems to start. They should work to find an equation for each of the patterns. 3. Tell them what materials are available for them to use. Remind them that they can also draw. 4. Let them know that they will have about 20 minutes to work the problems. 5. As they work the problems, walk around asking questions and helping participants. Be aware of the different strategies and equations participants are using so you can have a variety of presentations. 7. Hand out chart paper to each group. Assign one problem for them to present. Their presentation should include: <ol style="list-style-type: none"> a) their method for organizing information about the pattern b) a way to show the pattern that they found c) an equation or expression and how they found it d) any questions they have for the class (optional) 8. Begin the presentations with problem one, then two, etc. 	<p>The equations for numbers 1-3 are all linear. The others are not. Participants may be able to find the expressions for 4-6, but it takes more time and much more discussion.</p> <p>This type of problem solving works best when there is a variety of material for use. It is best to let participants choose their materials although sometimes it is necessary to say, <i>“Why don’t you use ... to help you figure this out.”</i></p> <p>If participants are “stuck”, it is sometimes helpful to have them visit another group that is working the same problem to get some ideas or clarify an issue.</p> <p>Remember that the expressions for numbers 4-6 are more difficult for the participants to find.</p> <p>See Notes for Equations and Problem Solving (Facilitator Notes on page 23) for additional information on the problems for both pages of this worksheet.</p> <p>Have the whole group go up to present. Try to have each member of the group talk but do not force them to do so. When groups present, have the class acknowledge them by clapping.</p>
Problem Solving Strategies (10-15 minutes)	
<ol style="list-style-type: none"> 1. Ask participants to think about the different ways the class solved each of the problems (what they did, what they used). Ask each table to write down their ideas. 2. Collect ideas from the class by asking each group to start by sharing one idea. Record their thinking on Chart It!. 3. Continue sharing until all strategies are shared. 4. After commenting on their ideas, tell them that you have a list of problem solving strategies to share. Distribute Problem Solving Strategies and display the transparency. 	

Activities

Problem Solving Strategies (10-15 minutes)	Notes
<p>5. The following questions are useful for discourse:</p> <ul style="list-style-type: none"> • Which of the strategies that are on the list did you use? • Are there strategies you could have used? For which problems? • Do you have any comments/questions about the strategies on the list? <p>6. Share with participants that these strategies are useful in solving problems not only for themselves but also for their children. Display the transparency, Bags of Cookies. Say:</p> <p><i>Suppose you are helping your child with the following problem:</i></p> <p>There are 24 cookies that need to be put in bags. Each bag will have 3 cookies. How many bags will there be?</p> <p>7. Ask:</p> <p><i>Which one of the problem solving strategies could your child use?</i></p> <p>8. If time permits, have participants try one or more of these strategies to solve the cookie problem.</p> <p>9. Tell the participants that we all have strengths and preferences with problem solving. Their preference may differ from their children's preference. Ask:</p> <ul style="list-style-type: none"> • Which of these strategies are you more comfortable with? • Which of these do you think your child is more comfortable with? • How could you help your child learn to use these strategies? 	<p>Asking the participants to think about the strategies that they used and having them develop a problem solving strategies list before giving them a list is important for their learning.</p> <p>Accept all of their ideas and record their strategies in their own words.</p> <div data-bbox="1149 548 1349 856" style="border: 2px solid black; padding: 5px; text-align: center;">  <p>Various strategies to solve problems: make picture, find a pattern, etc Chart It!</p> </div> <p>The list purposefully is not intended to be an exhaustive list of strategies.</p> <p>Using a problem from the adopted textbook can be quite effective. Getting several problems from various grade levels is even more effective.</p> <p>Some problem solving strategies for the cookie problem are: acting it out, using objects, drawing a picture, making an organized list</p>
<h3>Exploring Patterns (5-15 minutes)</h3>	
<p>1. Tell them that developing equations requires problem solving skills. We are going to practice problem solving with some more difficult problems. Tell them to choose one of the problems from Equations and Problem Solving, answer the questions, and describe the strategies that they used from the Problem Solving Strategies list. Then have groups share their strategies.</p> <p>2. Tell them that strengthening these problem solving strategies in your child will help them with their algebraic thinking.</p>	

Activities

Take Home Activities (5-15 minutes)	Notes
<p>1. Have participants work with their children on the handout Engaging Your Children in Problem Solving. Encourage them to ask their children to solve the problems in more than one way.</p> <p>2. Have participants complete any problems from Equations and Problem Solving that they did not complete.</p> <p>3. Display and hand out the Money Exchange Game. Model playing the game. Discuss the game in detail before the end of the session.</p> <ol style="list-style-type: none"> Ask for a volunteer and model playing the game on the overhead so everyone understands the rule. Have them play with one penny and one nickel. They are always successful at this. To help participants find a method of recording, ask: <ul style="list-style-type: none"> <i>What move did you just make?</i> <i>How can you record that?</i> Let them know that they should try the game with two coins on each side and then with three, recording their moves for each game. Ask them to look at their list of problem solving strategies. <p><i>What strategies can you use to figure out how to play this game with four coins on each side?</i></p> <p>Participants are now ready to tackle this problem at home.</p>	<p>The take home activity assigned will be dependent on the needs of the participants.</p> <p>The first suggested activity deviates from the topic, so parents can use the Problem Solving Strategies with their children.</p> <p>The Money Exchange Game is quite challenging for the participants. It is necessary to spend about 5-10 minutes making sure that the participants understand how to play the game. Some problem solving strategies that work for this game are: look for a pattern, make an organized list, and solve a simpler/similar problem.</p> <p>An interesting challenge is to use their pattern to record the steps for a game with five coins on each side and then play the game from the recording.</p>
<h3>Closure (5 minutes)</h3>	
<p>Ask participants what they have learned that was new for them about equations or problem solving.</p>	
<h3>Preparation for the next Session</h3>	
<ol style="list-style-type: none"> Collect name cards if applicable. Save the Chart It! and bring it to the next class. Optional: <ul style="list-style-type: none"> Type the notes on the Chart It! and distribute at the next session. Take digital pictures of each chart. Prepare handouts of these pictures. 	

Facilitator Notes

Notes for Equations and Problem Solving

Remember that all the problems do not need to be discussed unless participants worked on them. If finding the expressions for the handshake problem and the Twelve Days of Christmas problem are too difficult for them, let them know that they will see them again in a later session. (In session 7, they will graph them and compare their graphs.)

1. Matchstick Square

Materials to use: toothpicks, straws, or drawings

- The answer is some form of $3s + 1$
- Some see the pattern as adding three each time plus the original one at the end.
- 60 squares would need 181 matchsticks.

2. Banquet Tables

Materials: use pattern blocks for trapezoids

- The answer is some form of $3t + 2$
- Some think of it as 3 seated at each table plus the one at each of the ends which gives the expression, $3t + 2$;
- Some think of it as 4 at each table minus 1 for each table not at an end (the number of these tables is $t-2$) which gives the expression, $4t - (t-2)$

3. Growing Squares

Materials: can use tiles to make squares, drawings

- The answer is some form of $4s$
- The perimeter is always 4 times the length of the side

4. Twelve Days of Christmas

Materials: paper/pencil, cubes or other counting objects

- The answers are:
 - 12 gifts on the 12th day
 - total number of gifts for all 12 days is 98
 - use cubes to form the numbers into triangular numbers
- The expression is $n/2(n + 1)$; $n + 1$ represents the sum of the first number, 1, and the last number n ; $n/2$ gives the number of groups of $(n+1)$
- Good discussion:
 - adding the series of numbers from 1-12 using the strategy of $1 + 12$, $2 + 11$, $3 + 10$, $4 + 9$, $5 + 8$, and $6 + 7$ or 6 groups of 13; You can develop this by adding smaller series of numbers whose last number is even
 - additional discussion can follow on how this works for an odd number of numbers such as the sum of the numbers from 1 to 5; this gives 2 groups of 6 plus $\frac{1}{2}$ of a 6 or $2\frac{1}{2}$ groups of 6

5. Handshakes

Materials: people to act it out, drawings also work (many use the same kind of drawings as they did for the diagonals problem)

- The answer is some form of $\frac{1}{2}(n)(n-1)$
- Solving a simpler problem works well to find the pattern of the number of handshakes. The process is similar to the problem above. this time the numbers start at 1 and end at 99. This gives 49 sets of 100 plus the middle number of 50.

Facilitator Notes

Notes for Equations and Problem Solving (continued)

5. Handshakes (continued)
- c) Participants often see the following pattern. First person shakes 99 hands, 2nd person shakes 98 hands, 3rd-97 hands and so on. Thus, you are again adding a series of numbers but this time the numbers start at 1 and end at 99. This gives 49 sets of 100 plus the middle number of 50.
 - d) Another way to look at it is that each person shakes everyone's hands. That person cannot shake his/her own hand, thus each person shakes $n-1$ hands. But since there is a double count in handshakes, then you need to take half of the count.
6. Tower of Offices
Materials: drawings, or cubes
This problem gives the same numbers as the Twelve Days of Christmas.

Notes for Money Exchange Game

1. Remember that a coin can either slide one space or jump another coin. The pennies can only move in one direction and the nickels can only move in the other direction.
2. Participants have used various methods to record the steps.
 - a) One method to record this is using slide and jump.
 - b) A second method of recording is to record the coins being moved.
 - c) Other methods of recording are using arrows indicating slides and jumps.
3. Method of Recording using slide and jump and coins being moved
 - a) 1 coin per side with one space in the middle of them
Start with 1 coin on each side and 1 space in the middle of them.
Recording: slide, jump, slide OR
penny, nickel, penny.
 - b) 2 coins per side with one space in the middle of them
Recording: slide, jump, slide, jump, jump, slide jump, slide OR
nickel, penny, penny, nickel, nickel, penny, penny, nickel
 - c) 3 coins per side with one space in the middle of them
Recording: slide, jump, slide, jump, jump, slide, jump, jump, jump, slide, jump, jump, slide
jump, slide OR
P, N, N, P, P, P, N, N, N, P, P, P, N, N, P
 - d) 4 coins per side with one space in the middle of them
Recording: slide, jump, slide, jump, jump, slide, jump, jump, jump, slide, jump, jump, jump,
jump, slide, jump, jump, jump, slide, jump, jump, slide jump, slide