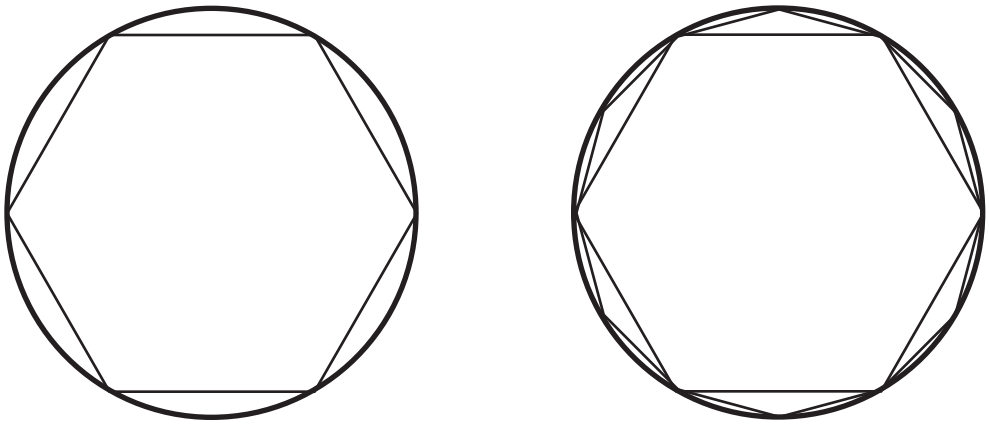
A "Thought" Experiment



(a) The initial hexagon (b) New vertices at the midpoints of the arches

Figure 1. Regular polygons that approximate a circle.

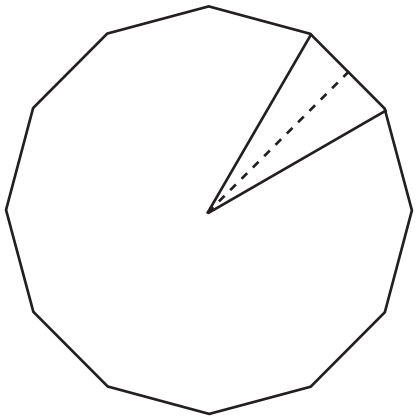


Figure 2. A triangle whose base is one side of the polygon.

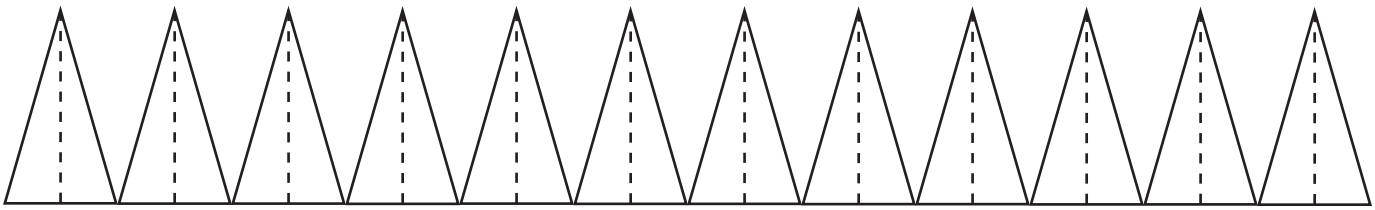


Figure 3. The polygon is broken into triangles.

We can also arrange the triangles that form the regular polygon into a parallelogram (see figure 4). Its base will be very close to half the circumference, $1/2 \times d \times \pi = 1/2 \times 2 \times r \times \pi$, that is $r \times \pi$, and its height will be very close the radius r of the circle. The area of the parallelogram will therefore be very close to πr^2. As the number of sides of the regular polygon increases, the height of the corresponding parallelogram gets closer and closer to the radius of the circle, and its base closer to πr. The area of the circle is given by $\pi r \times r = \pi r^2$.

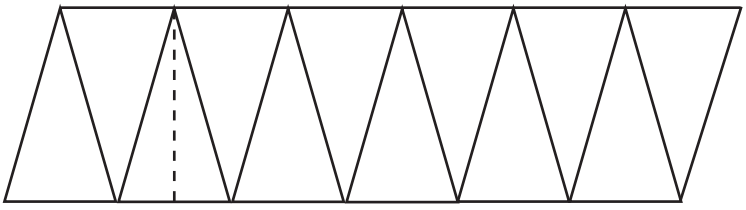


Figure 4. The polygon is rearranged into a parallelogram.