

Statistics

Children were given the following problem and asked what number belongs in the box.

$$8 + 4 = \square + 5$$

Percent of children who gave these answers:

| Grade * | Answers Given | | | | |
|---------|---------------|-----|-----|-----------|-------|
| | 7 | 12 | 17 | 12 and 17 | Other |
| 1 | 0% | 79% | 7% | 0% | 14% |
| 1 and 2 | 6% | 54% | 20% | 0% | 20% |
| 2 | 6% | 55% | 10% | 14% | 15% |
| 3 | 10% | 60% | 20% | 5% | 5% |
| 4 | 7% | 9% | 44% | 30% | 11% |
| 5 | 7% | 48% | 45% | 0% | 0% |
| 6 | 0% | 84% | 14% | 2% | 0% |

* Number of children questioned: Grade 1: 42; grade 1 & 2: 84; grade 2: 174; grade 3: 208; grade 4: 57; grade 5: 42; grade 6: 145.

Adapted from the Teaching Children Mathematics, December 1999
Article: "Children's Understanding of Equality: A Foundation for Algebra"

True or False

All of the statements below are true.
Here are some typical responses from children when they were asked if the statements were true or false:

| Problem | Typical response |
|-------------------|---|
| $3 + 4 = 7$ | It is true because when I add 3 and 4, I get 7. |
| $15 = 10 + 5$ | False. You can't do that! It's backwards. |
| $7 = 7$ | False. That doesn't make sense. There is nothing to do. |
| $20 - 7 = 17 - 2$ | False. That's not right! There are problems on both sides! |

Investigating Thinking

- Work in pairs.
- Look at one statement at a time.
- One person says if the statement is true or false.
- The second person practices asking questions to find out how the first person is thinking.

TRUE OR FALSE?

$$8 + 4 = 12$$

$$8 = 3 + 5$$

$$9 + 7 = 15$$

$$9 + 3 = 9 + 3$$

$$11 + 2 = 1 + 11$$

$$10 - 6 = 12 - 8$$

$$373 + 458 = 1481$$

$$2348 - 476 = 286$$

$$58 + 38 - 38 = 58$$

$$87 \times 1 = 88$$

Here are some questions to ask each other as you do these true/false statements:

- How did you decide whether the statement was true or false?
- What did you do first? Next?
- Did anyone do it differently?
- Could you have done it differently?
- What do you know about those numbers?
- How could you use manipulatives to show your thinking?

Problem Solving

Answer the questions below and explain how you got your answers:

1) $3 + 4 = 7$ True or False?
Why?

2) $15 = 10 + 5$ True or False?
Why?

3) $7 = 7$ True or False?
Why?

4) $20 - 5 = 17 - 2$ True or False?
Why?

5) $8 + 4 = \square + 7$ What number goes in the box: _____

At Home with the Equals Sign

Background Information

Questioning Skills

Research has shown that people learn by thinking and talking about their experiences. When you work with your child on homework, using the questions from tonight can help your child make sense of what they are doing and why.

Equals Sign

Children have a good understanding of the concept of equality. The concept that they misunderstand is the meaning of the equals sign. Most of the misconceptions about the equals sign come from the impression that students have that it means "to do something". This is not surprising when you think about how problems are set up in the early grades. It is not unusual for students to have a sheet of problems to do that all look like: $3 + 4 = \underline{\quad}$. One of the most important ways to help your child develop an understanding of the sign is to read it as "has the same value as" or "is the same as".

Here are some ideas for working with your child:

Ideas for Home

$7 = 7$

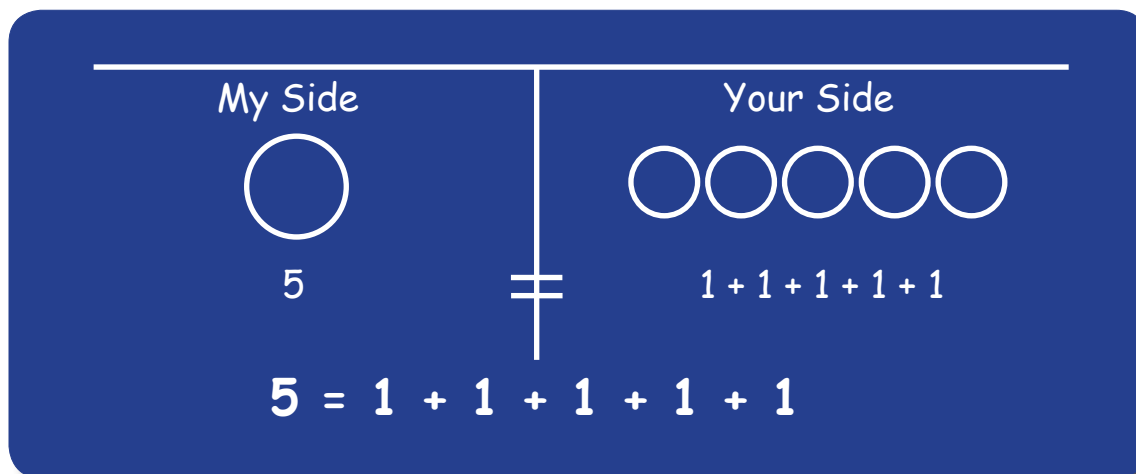
Some children believe this cannot be true because the equals sign means that they have to do something. To help them understand, take 7 M&M's (for instance), and ask them how many would they need to have the same amount? When they say 7, ask how they know. Then ask how they can put that in words. And finally, how can they record the information mathematically. (Sometimes they will record it as $7 + 0 = 7$. If this happens, ask them if you added anything to anyone's pile.) This gives you the opportunity to expand their thinking of what the equals sign means.

$15 = 10 + 5$

Some children believe that this problem is backwards, and therefore, cannot be done. You could use money to help with this concept. Start with a simpler problem such as $5 = 1 + 1 + 1 + 1 + 1$, instead of $15 = 10 + 5$. Get a paper and draw a line down the middle. Tell your child that the left side is your side and the right side is his/her side. Place a nickel on your side and a penny on their side and ask if you both have the same value. When they say no, say that you want everything to be the same value. Since this is not the same value, ask your child to add some more pennies until it is. After they have done so, record them on the bottom of the sheet: $1 + 1 + 1 + 1 + 1$.

At Home with the Equals Sign

Record the 5 for your side. When your child agrees that it is the same, tell them that it is time to write the sign that says it is the same: the equals sign. Have them write the equals sign between the two recordings. To reinforce this "backwards" statement, record it again at the bottom of the sheet. This idea of having a paper divided into two sides could be used for other examples. It resembles that idea of a balance and that both sides balance when you use an equals sign.



$$20 - 5 = 17 - 2$$

Students have a good understanding of equality. Build on that concept to show them that the two sides of a true/false question are equal if the values are equal. Manipulatives are good to use for this concept. Are 1 nickel and 5 pennies the same as 2 nickels? How would you record it? ($5+1+1+1+1=5+5$). Beans can also be used. make piles of 4 beans and 3 beans have as many total beans as a pile of 2 beans and a pile of 5 beans?. How do you know? How would you record the information mathematically? ($4 + 3 = 2 + 5$)

Ideas for True/False Statements

The true/false statements listed were designed to include:

- a standard looking question with the operation on the left: $8 + 4 = 12$
- a "backwards statement with the operation on the right: $8 = 3 + 5$
- a false statement: $9 + 7 = 15$
- a statement that has operations on both sides: $10 - 6 = 12 - 8$
- a false statement with large numbers that does not make sense if you estimate or round the numbers: $373 + 458 = 1481$ (Rounding the numbers up to the next hundred, I get $400 + 500 = 900$, so 1481 is too large.)
- a false statement with large numbers including subtraction: $2348 - 476 = 286$ (Rounding up, I get $2400 - 500$, or just about $2500 - 500$, which is 2000, so 286 is far too small.)

Use these ideas to draw up true/false statements for your child. It also might be fun to make up true/false statements with your child and enjoy the humor as you make up some of the false statements.

Instructional programs from prekindergarten through grade 12 should enable all students to--

**Represent and analyze
mathematical situations
and structures using
algebraic symbols**

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The Equate Game

GAME INSTRUCTIONS

Mathematics Focus: Algebraic thinking and number sense

Materials: Game board
 Game cards (cut out from the cardstock copies)
 A file folder or folded paper for each player (for hiding the cards from view)

Players: 2 or more (can be fun with teams) **Ages:** 6+

Instructions: The object of the game is to get the most points. Players use the game board in a similar fashion to scrabble, using numbers, operations and the equals sign in order create equations.

1. Players remove the equals sign cards and share them. They place all of the other cards face down.
2. Players draw to see who goes first (highest number goes first.)
3. Each player draws 10 squares and places them inside a folded paper so only he/she can see the cards.
4. Player one forms an equation with his/her cards. He/she scores one point for each card that he/she uses in the equation.
5. The player draws cards to replace all of the cards that he/she used during the turn.
6. The next player must use either a number, an operation or an equals sign that is already on the board.
7. The end of the game is when someone runs out of cards or no one has a way to play the rest of their cards.

Example:

I have drawn 1, 2, 4, 5, 8, 8, 7, +, -, ×

I can write the equation: $8 = 4 + 5 - 1$ (7 points)

Or I can write the equation $4 \times 8 - 17 = 8 + 7$ (9 points)

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| 8 | = | 4 | + | 5 | - | 1 |
| | | + | | | | + |
| | | 3 | | | | 5 |
| | | = | | | | = |
| | | 7 | | | | 6 |

The Equate Game

GAME BOARD

The first play must use the center double score. When a square has "Double Score" on it, that turn's total score is doubled. If it would have been worth 6 points, it is worth 12 points instead.

| | | | | | | | | | | | | |
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| | | Double Score | | | | | | | | Double Score | | |
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| | | | | | | | | | | | | |
| | | Double Score | | | | | | | | Double Score | | |
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The Equate Game

GAME CARDS

 Cut out game cards.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|------|------|------|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 5 | 5 | 6 |
| 6 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 8 | 8 |
| 8 | 8 | 8 | 8 | 9 | 9 | 9 | 9 | 9 | 9 | WILD | WILD | WILD |
| + | + | + | + | + | + | + | + | + | + | + | + | + |
| + | + | + | + | + | - | - | - | - | - | - | - | - |
| ÷ | ÷ | ÷ | ÷ | ÷ | ÷ | × | × | × | × | × | × | × |
| = | = | = | = | = | = | = | = | = | = | = | = | = |
| = | = | = | = | = | = | = | = | = | = | = | = | = |
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