## WORKSHOP EIGHT

## Patterns



| Blue | Blue | Red | Red | Blue | Blue | Red | Red | Blue | Blue |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Red | Blue | Blue | Blue | Red | Blue | Blue | Blue | Red |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Exploring Patterns and Repeating Units

## Outcomes

- To gain an awareness of patterns and relationships in today's classrooms.
- To experience hands-on patterning situations.
- To recognize the idea of patterns as a foundation for developing number sense.


## Overview

This module introduces the concepts of patterning in a fun and relaxed manner. After participants are welcomed and made comfortable, they explore with colored blocks, forming trains. The repeating patterns of the trains are discussed. Ideas are shared about recording the patterns and predictions for the next blocks in the trains. In this first exploration participants explore the same basic pattern with different color combinations, as well as different patterns using the same color combinations.

To extend the color patterns to nature, participants study and discuss the color patterns that are found on snakes. Facilitators can find more examples of snakes on the internet if they wish, using the instructions from the resource page. The snakes have color patterns, but the colors are not equal in length. Some participants describe the patterns using the relative lengths of the colors.

The third exploration with patterns is done with numbers. A favorite tool for looking at patterns is the 100 chart. Participants spend time exploring the chart for patterns. They discuss the reason the patterns occur with questions like "What can you say about numbers that are side by side in a row?" The also use the patterns to explore shortcuts for adding 9, 10 and 11 to any number.

At this point facilitators present ideas from the district curriculum related to patterns. Connections among the topics of this session, the curriculum samples, and the national mathematics standards are made.

Prizes for the estimation activity and take-home activities are distributed.

## Mathematics Background

The mathematical concepts in this module include:

- Exploring and recording patterns.
- Number relationships.

There are many opportunities to work with young people on patterning. Among these are color patterns, shape patterns, size patterns, rhythm patterns and number patterns. This workshop explores the surface of many of these patterning opportunities. The recording of a pattern with symbols helps to transfer ideas from a concrete level to the abstract level.

## Importance of patterns

Patterns are a way for young students to order and organize their world. The study of patterns in mathematics is important because it establishes habits of thought. Searching for patterns helps students discover the similarities that connect seemingly unrelated information. A student who expects to understand and make sense of mathematics looks for connections and patterns. These patterns make it possible to make predictions and check to see that procedures are logical. A student who does not see patterns often sees all concepts as separate and unrelated. They view mathematics as a set of rules to be memorized, making the understanding of mathematics disjointed.

## Using patterns to make predictions

In this workshop, one of the patterns that is explored is adding 10. Participants use the 100 chart to look at what happens to numbers as a person adds 10. As students recognize that adding 10 to 23 gives 33 , and that adding more 10 s gives $43,53,63,73$, they can use this pattern to make predictions. What if I had 37 and wanted to add 10 ? Can I make a reasonable prediction? What would happen if I added 20 instead of 10 ?

## Using patterns to make generalizations

To continue with this example, students can use their understanding about adding 10 to make generalizations. They might generalize that adding 10 changes the ten's digit, but does not change the one's digit. They can check their generalizations by adding 10 to many different numbers and checking the results. Understanding this pattern can lead to further generalizations relating to adding such numbers as 9 or 11. As students develop these generalizations, they start to see connections between seemingly separate operations.

## Room Setup

- Desks or tables arranged in groups of 4-6
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda
- Large area to hang butcher paper
- Space for participants to line up


## Materials

## Facilitator

## Transparencies

- Overhead projector
- Overhead pen
- Overhead square tiles (to match the colors of the colored cubes)
- Transparencies, write-on
- Chart paper
- Chart markers
- Butcher paper
- Masking tape
- Chips or beans
- Timer (optional)


## Participant

Individuals:

## Handouts

- Pencil
- Paper
- Colored markers or crayons
- Colored cubes (10 each of red and blue and 5 each of 2 yellow and green for each participant. Use $1 \times 1$ squares if colored cubes are not available.)
- Calculator
- Reflection

One per participant for class
BLM 88: Colored Train Patterns
BLM 91: 100 Chart
BLM 93: Helping My Child with Patterns
BLM 94: Pattern Sense
One per participant for home
BLM 96: Patterns at Home
BLM 97: 100 Chart at Home
BLM 98: What's My Pattern?

## Timing

2 hours

## Preparation and Timing (2 hours)

## Part 1: Preparing Classroom and Getting Started (10 minutes) - with children

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Make transparency of:
BLM 1: Welcome
Prepare before class for Part 1 and Part 2:
Colored cubes (10 each of red and blue and 5 each of yellow and green for each participant. Use 1x1 squares if
colored cubes are not available.)
Poster of agenda
Distribute to each participant:
Paper, pencils, colored markers or crayons, and Colored cubes
Make copies for each participant:
BLM 88: Colored Train Patterns
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Part 2: Setting the Stage (10 minutes)

No transparencies or handouts
Part 3: Building Train Patterns (15 minutes)

## Make transparency of: <br> BLM 89: Building Train Patterns <br> Part 4: Connections to Nature (10 minutes)

Make transparency of:
BLM 90: Snake Patterns (print in color)
Part 5: Patterns in the 100 Chart (15 minutes) - without children after \#4.

## Make transparency of:

BLM 91: 100 Chart
BLM 92: 100 Chart Activities
BLM 94: Pattern Sense
Make copies for each participant:
BLM 91: 100 Chart
BLM 93: Helping My Child with Patterns
BLM 94: Pattern Sense
Part 6: Connections (10 minutes)

## Make transparency of: <br> BLM 95: NCTM Algebra Standard

## Part 7: Take Home Applications (5 minutes)

## Make copies for each participant:

BLM 96: Patterns at Home
BLM 97: 100 Chart at Home
BLM 98: What's My Pattern?

## Part 8: Closing (5 minutes)

No transparencies or handouts
Distribute reflections or evaluations and estimation prizes

## Facilitator Resources

## Books

Standards 2000 Project, Principles and Standards for School Mathematics, The National Council of Teachers of Mathematics, Inc. (NCTM), 2000, p. 90, ISBN 0-87353-480-8, www.nctm.org

Math on Call: A Mathematics Handbook. Great Source Education Group, 1998, p. 465.
Turpin, Lorna. The Sultan's Snakes. Swindon, England: Child's Play, 1996

## Instructional Programs

Investigations in Number, Data, and Space, Kindergarten: Pattern Trains and Hopscotch Paths. TERC, Dale Symour Publications. 1998.

## Internet Resources

How to download images of pictures of snakes from the internet on a MAC:
Open Netscape Navigator or Internet Explorer and go to the Google search engine at http://www.google.com/. In the search box type in the name of the image you are searching for, i.e. pictures of snakes. Click on Images and then click on Google Search. After the images have loaded, click on the one you want and drag it to your document or desktop.

How to download images of pictures of snakes from the internet on a PC:
Open Netscape Navigator or Internet Explorer and go to the Google search engine at http://www.google.com/. In the search box type in the name of the image you are searching for, i.e. pictures of snakes. Click on Images and then click on Google Search. After the images have loaded, right click on the one you want and select "Save Picture As . . .". Save the picture where you like.

## Activities

## Preparation of Classroom

1. Before class begins, put colored cubes in sets. Each se $\dagger$ should have 10 of 2 colors and 5 each of 2 other colors for each participant. Use $1 \times 1$ squares if colored cubes are no $\dagger$ available.
2. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up estimation activities. Arrange desks or tables in groups of 4-6.
3. Display the transparency of BLM 1: Welcome!.

## Notes


4. Before participants arrive, have pencils, paper, color markers or crayons, set of color cubes, and copies of BLM88: Colored Train Patterns on the participant tables. Participants will build trains as they get settled.
5. Prepare and display a poster with the agenda and purpose of the session.

## Part 1: Getting Started (10 minutes) - with children

## Introductions

1. Introduce yourselves and then have the participants introduce themselves.
2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences.
3. Go over the agenda and purpose for the session.

## Part 2: Setting the Stage (10 minutes)

1. Have participants share what they have discovered while doing BLM-88: Colored Train Patterns. Record their responses on chart paper or a transparency. They may have noticed that the colors on train 1 and 2 are different, but they are arranged similarly. Trains 2 and 3 have the same colors, but different patterns. Write the words "repeating unit" on chart paper. Ask participants what this could mean. Lead them to a definition such as: The smallest part of a pattern that repeats itself. Say:

With your partner, discuss what the repeating unit is for each train.

Tell participants that the first and second trains have an $a-b-a-b-r e p e a t i n g$ pattern. The repeating unit is $a-b$.

## Activities

## Part 2: Setting the Stage (continued)

Ask:
What is the repeating pattern in the third train?
Tell the participants that when children are taught about patterns in school, they often identify the patterns using the ab notation. See Note A.
2. Explain that during this session, they will be exploring patterns.

## Notes

A. NOTE: In the train patterns, a could represent the first color and $b$ could represent the second color. If there were a third color, it could be represented by c. The letters could vary. For instance $r$ could be red and $b$ could be blue. Using symbols helps in writing patterns. Young children who do not write will symbolize patterns another way, probably with colored crayons.

## Part 3: Building Patterns (30 minutes)

1. Have participants disconnect their trains from the opening activity.
2. Ask participants to create a train that has a repeating unit of abb. Have one participants come up and display their train using transparency tiles.
3. Ask participants to create a train that has a repeating unit of acbc. Have one participants come up and display their train using transparency tiles.
4. Display BLM 89: Building Train Patterns while giving the instructions for the activity. This would be a good opportunity for parents to watch what their children create. Make sure that everyone has enough blocks for their trains.
a) Give participants 5-10 minutes to build their trains
b) Have them record their repeating units.


## Part 4: Connections to Nature (15 minutes)

Ask participants if there are animals that have patterns. Have them share their ideas. Tell them that snakes often have interesting patterns. Display BLM 90: Snake Patterns. Have the participants discuss the repeating patterns on the snakes in their groups. Ask for volunteers to share their ideas. Ideas may vary here. Most will see the patterns as color patterns (white, black, white, black) and some may add the dimension of length (white, black, black, white, black, black, seeing that the black is twice as long as the white). Both answers are valid. The length issue is not the focus of the activity, but if the participants mention it, the discussion would be enriched.

## Activities

## Part 5: Patterns in the 100 Chart ( 30 minutes) - without children after \#4

1. Start by saying:

We have been exploring patterns with colors and in nature. Now we are going to explore the rich world of numbers.

Display transparency BLM 91: 100 Chart. See Note B for ideas about using the transparency. Have participants look for patterns that they notice with the numbers. While participants are looking for patterns, distribute the handout of BLM 91: 100 Chart and chips or beans. Have participants share patterns that they have noticed. Point these patterns out as they are shared. Limit the sharing to about 3 ideas.
2. Display BLM 92: 100 Chart Activities. See Note $C$ for adjustments to the activities. Reveal one question at a time, recording participants responses below the instructions. Model the first moves of each instruction on the overhead transparency. Have participants remove the beans after each exploration.
3. Since this section is for families, let the parents know that this is an opportunity to learn more about their children's thinking. Their role is to observe. Say the following to the children:
a) Find all the numbers that contain at least one zero and cover them.

- What do you notice about them? (Accept all answers, recording them on chart paper or a transparency.)
b) Find all the numbers that contain at least one 3 and cover them.
- What do you notice about them? (Have them share)
- What makes them form a row? (Responses might include: each is one larger, the first number is always 3 - this is the tens place)
- What can you say about any numbers that are side by side in a row? (They are one apart)
- What can you say about one number that is directly below another number? (The top one is ten larger.)
c) Place a bean on 2, move 10 numbers and place another bean. Now move 10 more and place another bean. Continue through the chart.
- What do you notice?
- How is this similar to the other patterns?
- How is it different?


## Notes

BLM 91: Transparency
/ Handout


BLM 92: Transparency

B. NOTE:

Transparency Ideas:

- Write on a clear transparency laid over BLM 91: 100
Chart. Change it as often as you need to, keeping BLM 91 untouched.
- Use colored transparent strips to help identify the rows/ columns.
C. NOTE:

Adjustments to the 100 Chart Activities: - More advanced questions are called extensions.

- The number patterns can be simplified for younger children by using only 1 through 50.


## Activities

## Part 5: Patterns in the 100 Chart (continued)

## Notes

- What can you say about one number that is directly below another number?
- So, what happens to a number when you add 10 to it?
d) Find all the squares that contain two of the same number. (Model 11, 22)
- How is this shape different from the last pattern? (They form a diagonal.)
- How many squares it takes to move from 33 to 44.
(There are 11)
- What is the pattern for adding 11 to any number? (Move the tens up one and the ones up one.)
Try adding 11 to each of our numbers at the bottom of the transparency. Study the chart and see if you can find a pattern for adding 9. (Tens go up one while ones go down one.)

4. Use the following activities only if time permits. They are a part of the Take Home Applications from BLM 96:

## Patterns for Home.

Extension \#1:
Cover all the numbers whose digits add up to 10 .

- What do you notice about these numbers? (They should notice that the pattern is diagonal and that the numbers come in pairs, 1 with 9,8 with 2 , etc. The diagonal start with the 10, which is not part of the chosen numbers, but is the sum that you seek.)
- Are there any similar patterns with numbers? (The sums of 9, 8,7 , etc., all form similar diagonals.)


## Extension \#2:

Cover all the odd numbers that contain the number 3.

- How do you know if your numbers are odd?
- Does your rule work for all numbers? (They should come to the conclusion that if the ones digit is odd, the number is odd.)


## Extension \#3:

Find all the numbers whose digits add up to 15. (If necessary, model 78.) Participants should mark 69, 78, 87, 96.

- What do you notice about these numbers? (Some ideas might include that they come in pairs, 6 with 9 and 7 with 8 , or that the smallest number is 69 .)
- What would the next larger number be?
- Why is 69 the smallest number that we have?

Tell participants that they have done a very good job of finding patterns and that they can use the hundred chart at home to find more patterns. Have children leave. Then process with some questions about their child's thinking.

## Activities

## Part 5: Patterns in the 100 Chart (continued)

 - without children5. Tell participants that children do not naturally develop skills in adding tens and some of the other things that have come up in this workshop. Adding 10 can lead to adding multiples of 10 , such as 32 plus 20 to get 52 . Handout BLM 93: Helping My Child with Patterns. Ask:

When your child knows how to add 11, how might he/she add 21? (Skills like these help students to feel confident and curious about math.)
Have the participants reflect on what they have learned during this workshop about their child's thinking.

- What did your child understand?
- Where did he/she get stuck?
- What things do you want to remember to use to help your child?
Have participants share some of their observations about their children. Ask:

What things will make these explorations relaxed and fun for our children?
Brainstorm ideas and then emphasize that doing the activities together should be fun and light-hearted. Display transparency BLM 94: Pattern Sense and distribute the handout. Ask participants:

What do we want our children to know about patterns?
(Review the ideas on the transparency.)

## Part 6: Connections (10 minutes)

## Notes



BLM 94: Transparency / Handout


## Notes

1. Discuss what patterns look like in today's classroom. It is important at this time to bring in ideas from your district's curriculum that relate to patterns.
2. Display BLM 95: NCTM Algebra Standard to make connections among the topics of this session, your curriculum samples, and the national mathematics standards on pattern topics. Ask participants if they did all of the items listed on the standards during the session.
a) Understand patterns and relationships: They did the understanding patterns and relationships in all of the activities.
b) Represent situations using symbols: They were asked to record their patterns in the form of $a b$ or $a a b$, etc.
c) Use mathematical models: Models were used to build their trains.
d) Analyze change: They analyzed the changes in numbers from 22 to 32 and from 33 to 44 , etc. They also checked to see what was changing on the 100 chart when they moved from 33 to 44.

BLM 95: Transparency


## Activities

## Part 7: Take Home Applications (5 minutes)

1. Let participants know that there are many opportunities to encourage pattern thinking at home. Some of the ideas are written in the handouts.
2. Distribute handouts for home:

- BLM 96: Patterns at Home
- BLM 97: 100 Chart at Home
- BLM 98: What's My Pattern?

3. Participants can look for patterns in daily life.

## Notes

BLM 96: Handout


BLM 97: Handout


BLM 98: Handout


## Part 8: Closing (5 minutes)

1. If your district does not have an evaluation form to use, have them answer one of the following questions:

- What did you learn tonight?
- What will you do with your child as a result of this session?
- What did you find interesting tonight?

2. Distribute any prizes from estimations or drawings.
3. Thank participants for coming during their busy schedules.
