

Step by Step



Exploring Alternate Algorithms

Outcomes

- To demonstrate that many mathematical tasks can be accomplished using multiple approaches by providing parents with the opportunity to examine and discuss student thinking.
- To show how multiple approaches are connected.
- To inform parents of experts' opinions on the subject of the teaching of alternative algorithms.

Overview

This module is designed to demonstrate that many mathematical tasks can be accomplished using multiple approaches. It uses alternate algorithms for multiplication as its vehicle. Introducing parents to alternate algorithms serves to promote the notion that mathematics is creative and flexible rather than a set of strictly regimented procedures.

There is more reading in this workshop than in some of the others. If the participants are ESL parents or are weak in their reading skills, put them in groups with someone who has the ability to read for them.

In the beginning of this workshop, the participants brainstorm "recipes" for a fun birthday party. These "recipes" are discussed and used as a way to introduce the mathematical idea of algorithms. The section is designed to help participants understand what algorithms are and why it is advantageous to know more than one algorithm to produce a desired result.

Alternate algorithms are presented through examples of student work. This allows participants to engage in a collaborative problem solving process in order to uncover each student's thinking about the algorithm used. Active engagement in the process of making sense of a student's work is a deliberate goal of this section. Algorithms are examined one by one, without a rush to get through. The section provides an opportunity for parents to search for student thinking and will enable parents to engage in a similar practice when examining their own children's work.

The approach of teaching and encouraging alternate algorithms in the mathematics classroom often raises many questions in the minds of parents. This approach most likely was not part of their experience in school. For many adults, especially those who have had success in school mathematics, the notion of promoting alternative mathematical procedures makes them uncomfortable. The next activity presents experts' views on this topic in response to several common parent concerns. The intent of this section is to allay parents' fears that teaching children multiple approaches or encouraging invented strategies might be harmful to their children.

Several minutes are provided for making connections to the district's curriculum. There should be examples and explanations of how your district encourages alternate algorithms. Participants are also given time for processing the session's main ideas.

Several activities are provided for families to use at home.

Mathematics Background

The mathematical focus of this module is the exploration of alternative ways to do problems.

Algorithms

Most participants have learned to do problems in the traditional American algorithm. Although this method is traditional in the United States, it may not be the standard method in other countries. Learning many ways to do problems is not confusing for children. It gives them flexibility and tends to give them an understanding of how numbers and operations are related.

Using a traditional approach to mathematics, children are taught a single method for computing answers, without an explanation or analysis of why the method works to produce correct answers. Children are expected to master this procedure through memorization and repeated practice. This type of mastery is not connected to an understanding of operations, numbers, and their relationship.

Current thinking among mathematics educators offers a more meaningful approach to teaching computation. Experts suggest introducing children to many algorithms for a given operation. This method of teaching children to compute emphasizes the meaning of each operation and reinforces other important mathematical ideas such as place value, number sense, commutativity, mental strategies, etc.

Parents should not be concerned if their children experiment with inventing algorithms and testing them out. In fact, students who invent their own algorithms are demonstrating their ability to make sense of numerical situations and gaining confidence and competence in doing mathematics. (Burns, 1994)

Children who have access to several computational algorithms for an operation gain many benefits:

1. They are equipped to check their work with more accuracy;
2. They can choose the algorithm that best suits a given problem situation;
3. They can choose an algorithm they feel is most efficient and accurate for them.

Subtraction

Equal addends

There are two different approaches to subtraction. One is the concept of taking items away. An example would be that I have 24 apples and I take 18 away. How many are left? This approach is the one that we commonly use with young children as they first think about subtraction. With manipulatives, you take out 24 tiles and then take 18 away and count what is left. The second approach to subtraction is the concept of comparison or difference. I am 24 years old and my brother is 18 years old. What is the difference in our age? In this second problem, the two numbers are compared. With manipulatives, 24 red tiles can be set out and then 18 blue tiles. When the two sets of tile are paired up, there are 6 red tiles that do not have a partner, thus the 24 is 6 greater than 18. This second concept of comparing is very useful for the strategy of subtracting that is called equal addends. The equal addend method uses the addition of the same number to make the problem simpler.

$$\begin{array}{r} 24 + 2 \\ - 18 + 2 \end{array} \quad \text{becomes} \quad \begin{array}{r} 26 \\ - 20 \end{array}$$

The second problem is easier to solve and has the same difference as the first problem. It is as if you asked what would happen in 2 years? What will be the difference in our ages then? Difference remains the same through the years because an equal amount is being added to each age.

Room Setup

- Desks or tables arranged in groups of 4-6
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda

Materials

Facilitator	Transparencies
<ul style="list-style-type: none"> • Overhead projector • Overhead pens • Transparencies, blank • Chart paper • Chart markers • Masking tape • Timer (optional) • Scissors 	<p><i>BLM 1: Welcome</i></p> <p><i>BLM 33: Write a Recipe</i></p> <p><i>BLM 34: Examining Subtraction</i></p> <p><i>BLM 36: Examining Multiplication</i></p> <p><i>BLM 43: NCTM Communication Standard</i></p>
Participant	Handouts
<p>Individuals:</p> <ul style="list-style-type: none"> • Pencil • Paper • Scissors • Game board marker (beans, buttons, coins, etc.) • Reflection <p>Groups of 4-6:</p> <ul style="list-style-type: none"> • Butcher paper • Tape • Colored Markers • Bag of beans for each group (approx. 200) 	<p>One per participant for class</p> <p><i>BLM 35: Student Algorithms for Subtraction</i></p> <p><i>BLM 37: U.S. Standard Algorithm</i></p> <p><i>BLM 38: Area Method</i></p> <p><i>BLM 39: Partial Products</i></p> <p><i>BLM 40: Lattice Method</i></p> <p><i>BLM 41: Common Questions</i></p> <p><i>BLM 42: Experts' Responses</i></p> <p>One per participant for home</p> <p><i>BLM 44: Got It!</i></p>

Timing

2 hours and 10 minutes

Preparation and Timing (2 hours and 10 minutes)**Part 1: Getting Started (5 minutes) - with children****Distribute to each participant:**

Paper and pencils

Make transparency of:*BLM 1: Welcome***Part 2: Setting the Stage (15 minutes)****Make transparency of:***BLM 33: Write a Recipe***Part 3: Examining Student Work in Subtraction (40 minutes)****Make transparency of:***BLM 34: Examining Subtraction***Make copies for each participant:***BLM 35: Student Algorithms for Subtraction***Part 4: Examining Multiplication Algorithms (30 minutes)****Make transparency of:***BLM 36: Examining Multiplication***Make copies for each participant:***BLM 37: U.S. Standard Algorithm**BLM 38: Area Method**BLM 39: Partial Products**BLM 40: Lattice Method***Part 5: What do the Experts Say? (20 minutes) - without children****Make copies for each participant:***BLM 41: Common Questions**BLM 42: Experts' Responses***Part 6: Connections (10 minutes)***BLM 43: NCTM Communication Standard***Part 7: Applications for Home (5 minutes)****Make copies for each participant:***BLM 44: Got It?***Part 8: Closing (5 minutes) - with children****Distribute evaluations and estimation prizes**

Facilitator Resources

Articles

Broadbent, Frank W. “*Lattice Multiplication and Division.*” *Arithmetic Teacher*. January 1987. p.82-31.

Burns, Marilyn. “*Arithmetic: The Last Holdout.*” *Phi Delta Kappan*. February 1994. p. 471-476.

Caliandro, Christine Koller. “*Children’s Inventions for Multidigit Multiplication and Division.*” *Teaching Children Mathematics*. February 2000. p. 420.

Kamii, Constance, Barbara A. Lewis, and Sally Jones Livingston. “*Primary Arithmetic: Children Inventing Their Own Procedures.*” *Arithmetic Teacher*. December, 1993. p. 200 -203.

Leinwand, Steven. “*It’s time to abandon computational algorithms.*” *Education Week*. February 9, 1994. p. 6.

Simonsen, Linda M. and Teppo, Anne R. “*Using alternative algorithms with preservice teachers.*” *Teaching Children Mathematics*. May 1999. p. 516-519.

Stanic, George M. A., and William D. McKillip. “*Developmental algorithms have a place in elementary school mathematics instruction.*” *Arithmetic Teacher*. January 1989. p. 14-16.

Books

Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, p. 128, ISBN 0-87353-480-8, www.nctm.org

Morrow, Lorna J. (editor). *The Teaching and Learning of Algorithms in School Mathematics (1998 Yearbook)*. National Council of Teachers of Mathematics (NCTM). Reston, VA: 1998. ISBN 0-87353-440-9.

“*Whither algorithms? Mathematics educators express their views.*” p. 1-6

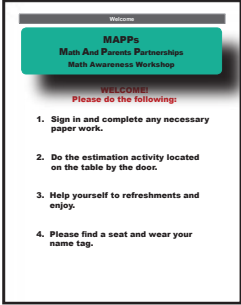
“*Children’s Invented Algorithms for Multidigit Problems.*”

“*The Harmful Effects of Algorithms in Grades 1-4.*”

Ma, Liping. *Knowing and teaching elementary mathematics: Teachers’ understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum. 1999.

Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell (editors). *Adding It Up : Helping Children Learn Mathematics*. Chapter 4: “*The Strands of Mathematical Proficiency.*” Mathematics Learning Study Committee, National Research Council. 2001. p. 115-156.

Activities

Preparation of Classroom	Notes
<ol style="list-style-type: none"> 1. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up estimation activities. Arrange desks or tables in groups of 4-6. 2. Display the transparency of BLM 1: Welcome!. 3. On the participant tables, distribute pencils, paper, butcher paper, tape, and colored markers. 4. You may want to have several puzzles on the tables for participants who arrive early. 5. Prepare and display a poster with the agenda and purpose of the session. 	<p>BLM 1: Transparency</p> 
Part 1: Getting Started (5 minutes) - with children	
<p>This module begins with parents and children. Children will be dismissed at the end of Part 4 of the workshop.</p> <p>Introductions</p> <ol style="list-style-type: none"> 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences. 3. Go over the agenda and purpose for the session. 4. Give an overview of the evening by letting participants know that they will be looking at student work to understand different approaches to problems, reading what researchers have to say about alternative approaches, and then playing a game that provides an opportunity to practice different approaches. Children can be included during the first parts, and then will be excused before the research is examined. 	
Part 2: Setting the Stage (15 minutes)	
<ol style="list-style-type: none"> 1. Make sure participants are seated in groups of 4 - 6 and that each group is composed of both children and adults. 	

Activities

Part 2: Setting the Stage (continued)	Notes
<p>2. Display BLM 33: Write a Recipe. The task for each group is to write a "recipe" for a fun birthday party for a child. If necessary, remind participants that recipes often contain lists of ingredients and procedures for producing a desired result. Encourage participants to be creative and assure them that there is not a <u>right</u> answer for this task. Let them know you expect groups to generate a variety of "recipes" and the discussion that will follow the task will benefit from their creativity.</p> <p>3. Ask each group to write their "recipe" on large paper and post it in the room for all to see.</p> <p>4. Process this portion of the session using these questions:</p> <p><i>What is the same about all the recipes?</i></p> <p>Responses are likely to include cake, games, friends, and decorations. Make a list ingredients and step-by-step procedures that they have in common.</p> <p><i>What is different about some recipes?</i></p> <p>Some differences might be that some are more detailed than others, ingredients may be different, and the format of recipes might be different.</p> <p>5. Point out that these "recipes" are all different, yet each produces the same results: a fun birthday party. Tell the participants that the title of this session is Step-By-Step. Let them know that in this session they will consider several step-by-step "recipes" for answering mathematical problems. Inform them that the word for a mathematical "recipe" is <u>algorithm</u>. This is likely an unfamiliar word for participants, so share with them this story of the origin of the word to help them better understand it.</p> <p>6. Optional: read "The Origin of the Word Algorithm" to the participants:</p> <p><i>The Origin of the Word ALGORITHM:</i> <i>Many centuries ago in Morocco there was a brilliant mathematician who was called, Al-Kwarizm. He studied step-by-step procedures related to astronomical topics. Today most languages around the world use his name to describe step-by-step procedures. In English we have the word "algorithm". In Spanish we have the word "algoritmo". Both have a pronunciation very similar to the Arabic name, Al-Kwarizm.</i></p>	<p>BLM 33: Transparency</p>

Activities

Part 2: Setting the Stage (continued)	Notes
<p>7. Tell participants that an algorithm is like a recipe. As they know, a cooking recipe gives step-by-step procedures to produce a certain food. A mathematical algorithm or "recipe" is simply a step-by-step procedure used to answer a problem.</p> <p>8. Explain to participants the reason for doing a Write a Recipe activity: Just as there are many approaches to planning a birthday party, there are many ways to accomplish mathematical tasks. Though they were likely taught one procedure for doing such things as multiplying, there are other processes for finding answers. Tonight's session will demonstrate several alternate algorithms for multiplication.</p>	
Part 3: Examining Student Work in Subtraction (40 minutes)	
<p>1. Tell participants that they will be given the opportunity to examine the work of some students who solved subtraction problems, but first it is important to look at subtraction more closely. Ask:</p> <p><i>Please do the following 2 problems and use the beans to show what is happening in them. Do the first problem. Leave your work on the table and get some more beans for the second problem.</i></p> <p>a) <i>I have eight beans. I give away five. How many do I have left?</i></p> <p>b) <i>I have eight beans and you have five. How many more do I have?</i></p> <p><i>Explain your thinking to others in your group.</i></p> <p>After participants have discussed and modeled their solutions with the beans, ask:</p> <p><i>We know these are both subtraction problems. What do they have in common? How are they different?</i></p> <p>Discussion of subtraction</p> <p>2. Explain that there are two forms of subtraction.</p> <p>a) The first is the idea of taking away. The first problem is an example of this form of subtraction. Participants start with a pile of 8 beans and then they remove 5. The answer, the difference, is what is left.</p> <p>b) The second is the idea of comparison. The second problem is an example of a comparison. In the comparison model, there are two piles of objects and these two piles are compared. In this problem, the pile of 8 has 3 more beans than the pile of 5.</p> <p>Students use both concepts when they invent their own methods (algorithms) for subtraction.</p>	

Activities

Part 3: Examining Student Work in Subtraction (continued)

3. Explain to participants that they will be given the opportunity to examine the work of some students who solved subtraction problems. Each student has used a different method (algorithm) for solving the problem. The participant's task is to examine each student's work and determine the step-by-step procedure used. Display **BLM 34: Examining Subtraction**. Read the directions aloud and make sure participants understand the task:

- Describe the step-by-step procedure this student used.*
- Try the procedure on the other three problems at the bottom of the example.*
- How is this method like/unlike the algorithm that you were taught? How is this method like/unlike the other algorithms on the sheet?*
- What might be some advantages of this student's method?*

4. Distribute **BLM 35: Student Algorithms for Subtraction**. This handout shows several methods that students used. Have the participants work on this sheet for 20 minutes, examining the student thinking and sharing with their groups.

5. Discuss each method. If possible, have a participant explain the student algorithm to everyone.

a) **Steve's Method:**

In this standard U.S. algorithm, students learn to exchange. Today we call it regrouping, but 20 years ago we called it borrowing. It is not possible to take 6 ones away from 4, so more ones are needed. If we exchange one of the tens for ones, we now have 14 ones and we can take 6 away.

b) **Jose's Method:**

In this method, José is adding an equal amount to the both numbers, the minuend and the subtrahend. He can do this because subtraction describes the distance from one number to the next. An everyday example of this is the idea of ages. If I am 16 and my sister is 10, the difference in our ages is 6. It will still be 6 when I am 20 and my sister is 14. As a matter of fact, it does not matter how many years go by, the difference in our ages will remain at 6. Students use this concept to make subtraction problems friendlier.

Notes

BLM 34: Transparency

Examining Subtraction

- Examining Student Work in Subtraction:
 - Describe the step-by-step procedure this student used.
 - Try the procedure on the other three problems at the bottom of the example.
 - How is this method like/unlike the algorithm you were taught? How is it like/unlike the other algorithms in this session?
 - What might be some advantages to this student's method?

BLM 35: Handout

Student Algorithms for Subtraction

Steve's Method

$$\begin{array}{r} 14 \\ - 8 \\ \hline 6 \end{array}$$

Jose's Method

$$\begin{array}{r} 16 \\ - 10 \\ \hline 6 \end{array}$$

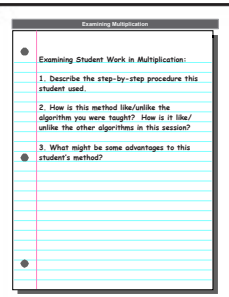
Another's Method

$$\begin{array}{r} 16 \\ - 10 \\ \hline 6 \end{array}$$

Another's Method

$$\begin{array}{r} 16 \\ - 10 \\ \hline 6 \end{array}$$

Activities

Part 3: Examining Student Work in Subtraction (continued)	Notes
<p>c) Evie's Method: Evie actually using a similar method to Juan. She adds 10 to the minuend and the subtrahend when she adds 10 to 4 to get 14 and 10 to 26 to get 36. Participants will think that this method looks similar to regrouping that we do in the standard U.S. algorithm, except that the "regrouping" happens in the subtrahend instead of the minuend.</p> <p>d) Amber's Method: Amber subtracts what she can from the number and then adjusts. She appears to start from the left instead of the right, subtracting 20 from 70 to get 50. She then subtracts 4 from 6 and realizes that if she does that she has 2 more left to subtract. She subtracts the 2 from the 50 to get 48. She knows that she still has 2 left to subtract.</p> <p>6. After all of the methods have been discussed, ask participants which method really surprised them. Have them share their thoughts.</p> <p>7. Ask participants what the methods have in common. Responses could include: they all needed to address the fact that you cannot take 6 from 4. There were many ways to solve the problem including adding on to both numbers, exchanging a 10 for 10 ones, using the thought of 2 left to subtract. They could even use the idea of negative numbers if they understand them.</p>	
Part 4: Examining Multiplication Algorithms (30 minutes)	
<p>1. Explain to participants that they will be given the opportunity to examine the work of some students who solved multiplication problems. Each student has used a different method (algorithm) for solving the problem. The participant's task is to examine each student's work and determine the step-by-step procedure used. Display BLM 36: Examining Multiplication. Read the directions aloud and make sure participants understand the task:</p> <ol style="list-style-type: none"> <i>Describe the step-by-step procedure this student used.</i> <i>How is this method like or unlike the algorithm you were taught?</i> <i>How is it like or unlike the other algorithms show in this session?</i> <i>What might be some advantages to this student's method?</i> 	<p>BLM 36: Transparency</p> 

Activities

Part 4: Examining Multiplication Algorithms (continued)

2. Distribute **BLM 37: US Standard Algorithm**. This handout shows the US Standard Algorithm. **See Note A** for a summary of this method. This is likely the multiplication method most people in the room were taught in school. You do not need to spend much time explaining this one. It is provided as the basis for discussion of other methods.

3. Distribute **BLM 38: Area Method**. **See Note A** for a summary of this method. Return to **BLM 36: Examining Multiplication** and remind participants that their task is to make sense of this student's method. Encourage discussion and observations about the method. Ask:

- *Where did the 800 come from?*
- *Why is there a 24 in the bottom right corner?*
- *What similarities are there between this method and our US standard algorithm?*
- *Which numbers are related in the two methods?*
- *Why would the second method be called the area method?*

Once the method is understood, have participants try the extra problems on the page.

4. Distribute **BLM 39: Partial Products**. **See Note A** on next page for a summary of this method. Return to **BLM 36: Examining Multiplication** and remind participants that their task is to make sense of this student's method. Encourage discussion and observations about the method. Encourage comparisons of this method to the Area Method and the US Standard Algorithm. Ask:

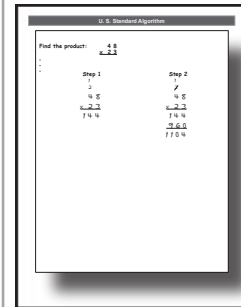
- *How did the student arrive at the 24?*
- *How did the student get 120?*
- *How about 160 and 800?*
- *What similarities do you see between this method and either of the other two methods?*
- *Why do you suppose this method is called the partial products method?*

Once the method is understood, have participants try the extra problems at the bottom of the handout.

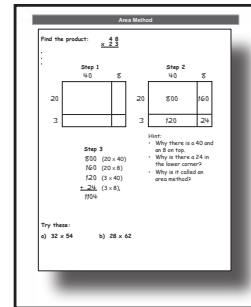
5. Some textbooks use the lattice method and parents have questions about it. If this is true in your district, do the following activity; otherwise, skip this section. Distribute **BLM 40: Lattice Method**. **See Note A** on next page for a summary of this method. Return to **BLM 36: Examining Multiplication** and remind participants that their

Notes

BLM 37: Handout



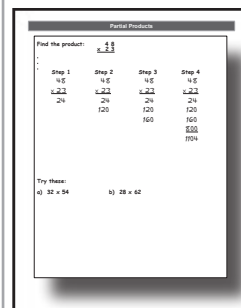
BLM 38: Handout



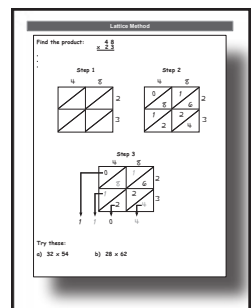
A. NOTE: Multiplication algorithms

- **BLM 37: US Standard Algorithm:** This is the method commonly taught in US classrooms. Each digit in the lower number is multiplied by each digit in the upper number. When a product of two digits produces a two-digit number, the tens digit is regrouped ("carried") to the next multiplication.
- **BLM 38: Area Method:** In this method, multiplication problems are represented visually as rectangular regions. The area of each smaller region is found then combined to find the total. It is related to the Partial Products method (BLM 40). Draw a rectangle in which the length and width represent the two factors. Draw lines to divide the rectangle showing the place value of each factor. Find the area of each of the interior rectangles, then add these areas together.

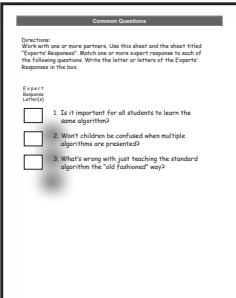
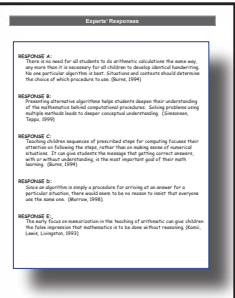
BLM 39: Handout



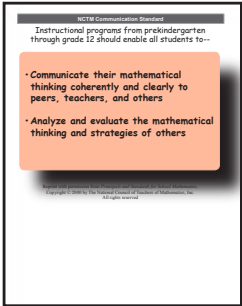
BLM 40: Handout



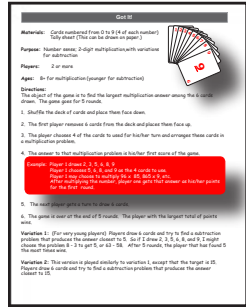
Activities

Part 4: Examining Multiplication Algorithms (continued)	Notes
<p>task is to make sense of this student's method. Encourage discussion and observations about the method. Encourage comparisons of this method to the Rectangle Method, US Standard Algorithm, and Partial Products. Once the method is understood, pose this multiplication problem for all to solve using Lattice Method: 37×41.</p> <p>6. Ask participants if they have another way that they do the problem. Have them share their method.</p> <p>DISMISS CHILDREN TO CHILD CARE AT THIS POINT.</p> <p>7. Re-seat participants if necessary so that no one is seated alone now that the children have been dismissed.</p>	<p>A. Note continued</p> <ul style="list-style-type: none"> • BLM 39: Partial Products: In this method, no regrouping (carrying) is required. Four multiplications are performed while keeping place value in mind. Multiply each pair of factors as in the standard algorithm, but write the entire product (no carrying). With each multiplication, the place value of each digit is maintained. Add these products. • BLM 40: Lattice Method: In this method the products of pairs of digits are found and recorded in a "lattice" frame. The lattice design accounts for the place value of each recorded digit. Write the digits of the factors along the top and right side of the lattice diagram. Multiply each pair of factors and write the product as a two-digit number (tens over ones) in the corresponding cell. Add along diagonals starting with the right most diagonal. If necessary, regroup (carry) from one diagonal to the next.
Part 5: What do the Experts Say (20 minutes) - without children	
<p>1. Explain to the parents that they will now complete an activity with the adults they are seated with. This activity will inform them of additional expert opinions on the subject of the teaching of alternative algorithms.</p> <p>2. Distribute BLM 41: Common Questions and BLM 42: Experts' Responses. Read the directions aloud to participants:</p> <p><i>DIRECTIONS: Work with a partner. Use this sheet and the sheet titled "Experts' Responses". Match one or more expert responses to each of the following questions. Write the letter or letters of the Experts' Responses in the box.</i></p> <p>Let participants know there will be more than one Expert Response for some questions. Allow 10 minutes for this task.</p>	<div style="display: flex; justify-content: space-around;"> <div data-bbox="1003 1297 1247 1633"> <p>BLM 41: Handout</p>  </div> <div data-bbox="1273 1297 1516 1633"> <p>BLM 42: Handout</p>  </div> </div>

Activities

Part 5: What do the Experts Say (continued)	Notes
<p>3. After participants have completed the task, ask groups to share the letters of responses they associated with each question. Encourage discussion of the questions and the expert responses. Participants are likely to associate responses as follows, but accept all participant input for this activity:</p> <ul style="list-style-type: none"> a) Question 1 relates to Responses A and D. b) Question 2 relates to Response B. c) Question 3 relates to Responses C and E. <p>4. Process this activity using the following prompts:</p> <ul style="list-style-type: none"> • <i>What is something you learned from the experts that surprised you?</i> (Explain) • <i>What is something you learned from the experts that reinforced your own ideas?</i> (Explain) • <i>What is something you learned from the experts that you are still wondering about?</i> (Explain) 	
Part 6: Connections (10 minutes)	
<p>1. Review the algorithms of the session.</p> <p>2. Tell participants that they have been communicating about mathematical ideas and that the NCTM standards talk about the need for communication in the classroom. Display BLM 43: NCTM Communication Standard. Ask them how creating and discussing strategies and different algorithms reinforces this standard. Have participants share their ideas.</p> <p>3. Connect the algorithms to what is happening in the classroom in your district. Are children inventing algorithms? Are they given a choice of algorithms? What choices are they given? Have some examples from the district's adopted text available for parent to see.</p> <p>4. Remind the participants that when children have an assortment of algorithms that they can use, they become more mathematically powerful. Remind them that the theme of the session is exploring different approaches to problems so that their children will have many options at their disposal when they try to solve problems. When students have a deep understanding of math concepts, they perform better on standard testing. We teach algorithms for efficiency: Are we teaching math or efficiency? A method that is efficient for one person is not necessarily efficient for the next person. As students understand multiplication, their methods become efficient.</p>	<p>BLM 43: Transparency</p>  <p>The image shows a transparency of the NCTM Communication Standard. It has a title 'NCTM Communication Standard' at the top. Below it, a subtitle reads 'Instructional programs from prekindergarten through grade 12 should enable all students to--'. There are two bullet points in a red box: 'Communicate their mathematical thinking coherently and clearly to peers, teachers, and others' and 'Analyze and evaluate the mathematical thinking and strategies of others'. At the bottom, it says 'Copyright © 2000 by the National Council of Teachers of Mathematics, Inc. All rights reserved.'</p>

Activities

Part 6: Connections (continued)	Notes
<p>5. Read this quote to the participants and tell them that this is the opinion of a mathematics education expert named Lorna Morrow. The complete reference for this quote can be found on the Facilitator Resources page of this module.</p> <p><i>“It is important for students to practice what they learn. However, this practice can take many forms. The use of arithmetic games and computer programs are two alternative ways teachers can provide practice.”</i></p>	
Part 7: Applications for Home (5 minutes)	
<p>1. Say:</p> <p><i>One way to support your child at home is to try to understand his/her thinking. When your child approaches a problem in a way you are not familiar with, probe their thinking to understand and support their logic. Keep in mind what you learned tonight about the value to students of using a variety of approaches--recipes for success.</i></p> <p>2. Introduce a game that participants can play with their children. Distribute BLM 44: Got It! and explain the game to the participants. There are three variations in order to accommodate the various ages and abilities.</p> <p>3. If available, hand out games from the district's curriculum materials for parents to play with their children at home.</p>	<p>BLM 44: Handout</p>  <p>Materials: Cards numbered from 1 to 10 of each number Tally sheet (This can be drawn on paper)</p> <p>Purpose: Number sense, 2-digit multiplication with variations for addition.</p> <p>Player: 2 or more</p> <p>Age: 8+ for multiplication (easier for addition)</p> <p>Directions: The object of the game is to find the largest multiplication answer using the 6 cards drawn. The game goes for 6 rounds.</p> <ol style="list-style-type: none"> 1. Shuffle the deck of cards and place them face down. 2. The first player removes 6 cards from the deck and places them face up. 3. The player chooses 4 of the cards to use for their turn and arranges these cards in a multiplication problem. 4. The player to their right looks at the problem to check their score of the game. <p>Example: Player 1 chooses 4, 5, 6, 8, 9, 10 Player 2 chooses 2, 3, 4, 5, 6, 8 Player 3 chooses 1, 2, 3, 4, 5, 6 Player 4 chooses 1, 2, 3, 4, 5, 6 Player 5 chooses 1, 2, 3, 4, 5, 6 Player 6 chooses 1, 2, 3, 4, 5, 6</p> <p>5. The next player goes back to draw 6 cards.</p> <p>a. The game is over at the end of 6 rounds. The player with the largest total of points wins.</p> <p>Variation 1: (For very young players) Players draw 6 cards and try to find a subtraction problem that produces the answer. Player 1's, 20 - 2 = 18, 2, 5, 4, and 5. Eight. Check the numbers 8 - 3 to get 5, or 45 - 36, after 6 rounds, the player that has found 5 the most times wins.</p> <p>Variation 2: This version is played similarly to variation 1, except that the target is 15. Player 1's 15 - 5 = 10, and try to find a subtraction problem that produces the answer.</p>
Part 8: Closing (5 minutes) - with children	
<p>1. If your district does not have an evaluation form to use, have them answer one of the following questions:</p> <ul style="list-style-type: none"> • <i>What did you learn tonight?</i> • <i>What will you do with your child as a result of this session?</i> • <i>What did you find interesting tonight?</i> <p>2. Distribute any prizes from estimations or drawings.</p>	

