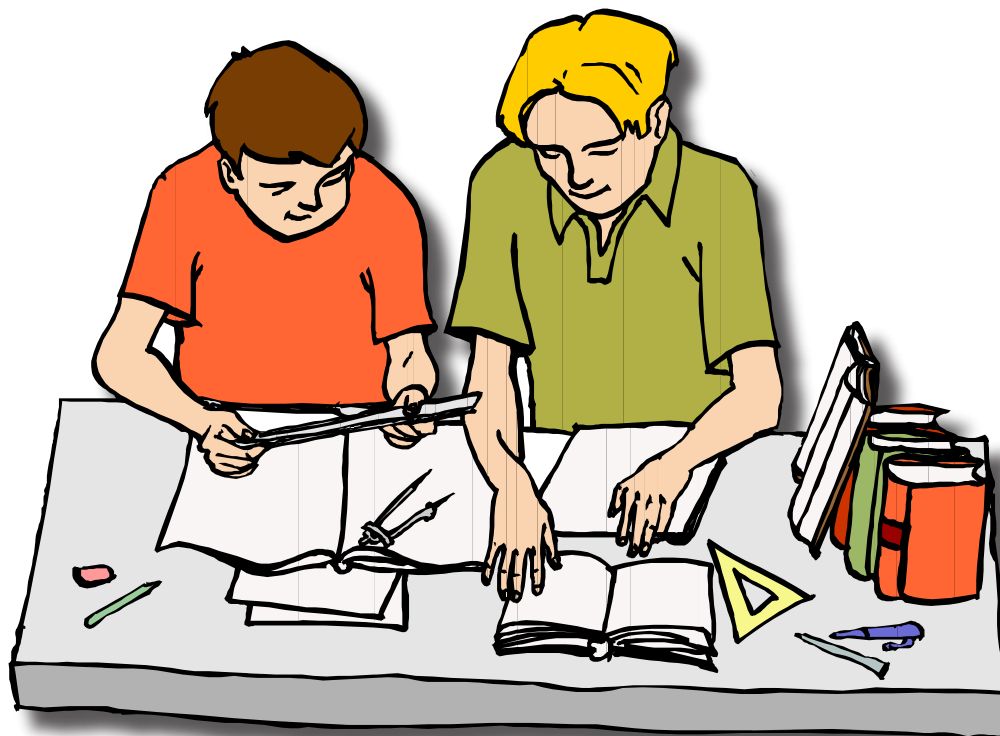


Problem Solving



Logic and Reasoning

Outcomes

- To explore skills employers desire in employees.
- To explore how these skills are reflected in how students are learning mathematics.
- To explore multiple strategies for the same problem to understand how differently people think about the same problem.

Overview

The main ideas of this workshop are working as a team to solve a problem and sharing strategies. In order to establish the importance of these skills, participants work on the activity named Fortune 500. The idea came from an article from Fortune 500. In it, major companies were asked to rank qualities that they desire in prospective employees. Teamwork, problem solving, interpersonal skills and communication skills were ranked as the top skills desired by employers.

After discussing the qualities from the Fortune 500 activities, participants solve a problem as a team. There are two options for the problem. The first is a pizza problem: "How many pizzas do the people at your school eat in a year?" This question is used with families in the workshop. The second question should be used when only parents are in the workshop, as it is not accessible to children. It was used in an interview for a major high-tech company: "How many gas stations are there in Tucson?" There are some thoughts on these problems in the mathematical background.

There is ample time in the session for teams to make posters and share the strategies and assumptions that they made when doing the problem. Sometimes there are unexpected connections between the teams and how they approached the problem. It is stimulating for the participants to see these connections. The richness in this activity are the many approaches to the problem.

Connections are made between the problem-solving experience, the national standards, and the skills from the Fortune 500 activity.

The next section of the session is important for making connections to the district's curriculum. It should be supplemented with examples and explanations of how your district encourages cooperative learning and problem solving.

The session ends with prizes for the estimations and the district evaluation or a final reflection.

The parent reference, Figure This!, must be ordered with ample time if it is to be used in Part 6. Ordering information can be found on **BLM 51: Resources**.

It is important to work the problems ahead of time in order to be aware of the challenges that they offer for participants. The workshop suggests using the problem, How many fingers are in this room?, as a mode. Organize this problem in a poster. The lettering in the poster needs to be large enough for all to see (modeling how big they will need to make their writing on the poster). The poster should state the problem, and then have a section for assumptions. Since the poster will be made ahead of time, you will not know how many people will be attending the session. Assumptions would include how many fingers each person has, and how many people might be attending the workshop. Your assumption will be wrong, but it will be based on logical thinking. You can explain that thinking during the workshop. Write "my method", then do the mathematics of the problem: 10×18 (or however many people you assume will be in the workshop). Finish the problem by listing the answer under a section called "my solution".

Mathematics Background

The focus of this module is problem solving and teamwork.

Connections to Principles and Standards for School Mathematics

The National Council of Teachers of Mathematics includes problem solving as a standard to be taught in all instructional programs, starting at the prekindergarten level. Problem solving should enable all students to:

- a) Build new mathematical knowledge through problem solving.
- b) Solve problems that arise in mathematics and in other contexts.
- c) Apply and adapt a variety of appropriate strategies to solve problems.
- d) Monitor and reflect on the process of mathematical problem solving.

Posing problems comes naturally to young children: They wonder about things like how long it would take to count to a million, or how many soda cans would it take to fill the school building. Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds. Teachers play an important role in the development of students' problem-solving dispositions by creating and maintaining classroom environments in which students are encouraged to explore, take risks, and share failures.

Also, the standards of reasoning and proof (developing and evaluating mathematical arguments), communications (communicating mathematical thinking coherently and clearly to peers, teachers and others), and connections (opportunities for students to experience mathematics in context, applied to their daily lives) are all integral parts of this module.

Team Work

In order to make connections between the NCTM Principles and Standards and the work world, the participants work on the activity named Fortune 500. The idea came from an article from Fortune 500. In it, major companies were asked to rank qualities that they look for in prospective employees. Teamwork, problem solving, interpersonal skills and communication skills are ranked as the top skills desired by employers.

Assumptions

In each of the problems there is ambiguity. Ambiguity leads to open discussions about the thought process that was involved in solving the problem. One of the processes is assumptions. Assumptions are the estimations of facts that are needed in order to solve the problem. With the pizza problem, participants need to make assumptions about how much pizza each individual eats in a year and how many people are in the school. Since there is no information on these exact numbers, participants have the opportunity to use logical thinking to estimate this information, and then discuss how they decided on the different assumptions. This leads to rich mathematical dialogue.

The Problems

Problem 1: Fill 'er up, please!

How many gas stations are there in our city? (This question can be changed to state or country.)

Participants have found a variety of ways to solve this problem. Some have looked at main crossroads and assumed an average of 1, 1.5, or 2 gas stations at these crossroads. Others have estimated the number of cars and assumed that a gas station is needed for every 3,000 to 5,000 cars. In one workshop, a parent owned a station and took the number of gallons that he typically sold each week, decided how many cars that would be, and then estimated the number of cars needed for the population. From there he determined how many gas stations would be needed to

Mathematics Background continued

service that number of cars. Surprisingly, the teams at this workshop were very close in their answers even though their approaches were quite different. It was quite a surprise to everyone. Whatever the method used or the answer, the richness of the question is in the diversity of strategies and the opportunity to share them. As the participants do their walk-about, they can pay attention to the team's method for organizing their thoughts and the assumptions that were made. If a team finishes early, an extension question could be: How much money do residents of your city spend on gas each week? Month? Or year?

Problem 2 That's a lot of Pizza!

How many pizzas are eaten by the people at your school in one year? There is ambiguity to this problem on purpose. The first question that will probably come up is: What size is the pizza? You can handle this by asking it before everyone starts and having everyone agree on the size, or you can let the size be one of the assumptions that the teams are making. Another question is: Who is included at the school? The intent was everyone. This makes the problem richer because some teams will include the principal, the janitor, the secretary and the bus drivers and some will not. Whatever assumptions they make, and whatever the answer, the richness of the question is in the diversity of strategies and the opportunity to share them. Since there is no "right" answer, each team's thinking can be celebrated and discussed openly. As the participants do their walk-about, they can pay attention to the team's method for organizing their thoughts and the assumptions that were made. If a team finishes early, an extension question could be: How much money do the people of your school spend on pizza in a year?

Room Setup

- Desks or tables arranged in groups of 4
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda

Materials

Facilitator	Transparencies
<ul style="list-style-type: none"> • Overhead projector • Overhead pens • Transparencies, blank • Chart paper • Chart markers • Masking tape • Timer (optional) • Estimation questions (prepared by facilitator) • Inexpensive prizes 	<p><i>BLM 1: Welcome</i></p> <p><i>BLM 39: Fortune Magazine Cover</i></p> <p><i>BLM 41: Fortune 500 Companies</i></p> <p><i>BLM 42: Problem Solving</i></p> <p><i>BLM 43: NCTM Problem Solving Standard</i></p> <p><i>BLM 44: Assumption</i></p> <p><i>BLM 45: Fill 'er Up, Please! or</i></p> <p><i>BLM 46: That's a Lot of Pizza</i></p> <p><i>BLM 47: Poster Instructions</i></p> <p><i>BLM 48: Walk About Instructions</i></p>
Participant	Handouts
<p>Individuals</p> <ul style="list-style-type: none"> • Paper • Pencil • Calculators • Post-it notes • Envelopes with Puzzle for Home (prepared by facilitator) • Reflection <p>Groups</p> <ul style="list-style-type: none"> • Chart paper • Chart markers 	<p>One per participant for class</p> <p><i>BLM 40: Preparing Skill Strips (copy, cut and prepare)</i></p> <p><i>BLM 41: Fortune 500 Companies</i></p> <p><i>BLM 43: NCTM Problem Solving Standard</i></p> <p>One per participant for home (make packet for easy distribution)</p> <p><i>BLM 49: Instructions for Puzzle for Home</i></p> <p><i>BLM 50: Puzzle for Home (copy on cardstock, cut and prepare)</i></p> <p><i>BLM 51: Resources</i></p>

Timing

2 hours

Preparation and Timing (2 hours)

Part 1: Getting Started (10 minutes)

Display transparency from workshop one:

BLM 1: Welcome

Part 2: Fortune 500 (30 minutes)

Make transparency of:

BLM 39: Fortune Magazine Cover

BLM 41: Fortune 500 Companies

Make copies for each participant:

BLM 40: Preparing Skill Strips (copy, cut and prepare)

BLM 41: Fortune 500 Companies

Part 3: Problem Solving (40 minutes)

Make transparency of:

BLM 42: Problem Solving

BLM 44: Assumption

BLM 47: Poster Instructions

Choose appropriate question for workshop and make transparency of:

BLM 45: Fill 'er Up, Please! or

BLM 46: That's a Lot of Pizza

Make copies for each participant:

BLM 43: NCTM Problem Solving Standard

Part 4: Walk About and Discussion (30 minutes)

Make transparency of:

BLM 48: Walk About Instructions

Part 5: Connections (5 minutes)

Make transparency of:

BLM 43: NCTM Problem Solving Standard

Part 6: Take Home Applications (5 minutes)

Make copies and prepare a take home packet for each participant:

BLM 49: Instructions for Puzzle for Home

BLM 50: Puzzle for Home (copy on cardstock, cut and prepare)

BLM 51: Resources

Part 7: Closing (5 minutes)

No handouts or transparencies

3-4 Inexpensive prizes for Estimation Question winners

Reflection / evaluations (*provided by the evaluation team*)

Facilitator Resources

Articles

Mikusa M, *Problem Solving is More Than Solving Problems*, Mathematics Teaching in the Middle School, September 1998, P. 20.

Van Zoest L , and Enyart A , *Discourse, Of Course; Encouraging Genuine Mathematical Conversations*, Mathematics Teaching in the Middle School, November-December 1998, P. 150.

Fortune Magazine Cover, Fortune, Vol. 147, No. 1, January 20, 2003, www.fortune.com/

The Top 500 Companies, Fortune, [www.fortune.com/fortune/fortune 500](http://www.fortune.com/fortune/fortune%20500)

Lost in Mathland — California's Controversial New Curriculum, Sacramento News & Review , Vol. 9, No. 2, April 27, 1997, www.newsreview.com

Books

Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, P. 256, ISBN 0-87353-480-8, www.nctm.org

Kagan, Dr. Spencer, Cooperative Learning, *Thinking Skills Structure*, Resources for Teachers, Inc., 1-800-Wee Co-op. 1994, P. 11:8, ISBN 1-879097-10-9, www.KaganCoopLearn.com

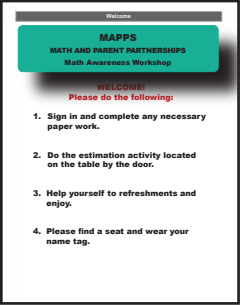

Internet Sites for Parents to Explore

Site addresses updated 9/21/03

Figure This!: <http://www.figurethis.org/>

Math Forum: <http://mathforum.org/>

Activities

Preparation of Classroom	Notes
<ol style="list-style-type: none"> 1. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up estimation activities. Arrange desks or tables in groups of 4-6. 2. Display the transparency of BLM 1: Welcome!. 3. Prepare and display a poster with the agenda and purpose of the session. 4. Prepare a poster of the sample problem: <ul style="list-style-type: none"> • How many fingers are in this room? Follow the format that is given to the participants: <ul style="list-style-type: none"> • What is the problem? What are your assumptions? • What is your method? • What is your solution? For a further discussion of this poster, see the overview. 	<p>BLM 1: Transparency</p>  <p>The transparency shows a green header with 'Welcome' and 'MAPPs MATH AND PARENT PARTNERSHIPS Math Awareness Workshop'. Below is a 'WELCOME!' section with a red line and the text 'Please do the following:'. It lists four instructions: 1. Sign in and complete any necessary paper work. 2. Do the estimation activity located on the table by the door. 3. Help yourself to refreshments and enjoy. 4. Please find a seat and wear your name tag.</p>
Part 1: Getting Started (10 minutes) - with children	
<p>Introductions</p> <ol style="list-style-type: none"> 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPs program. Have participants who are involved in the program share their experiences. 3. Give participants an overview of the session. Review the agenda and purpose of the session. When discussing the agenda, let the participants know the plan for including children in the session. 	
Part 2: Fortune 500 (20 minutes)	
<ol style="list-style-type: none"> 1. Set the stage by displaying the BLM 39: Fortune Magazine Cover transparency of the Fortune magazine and asking the question: <p><i>We hear a lot about what skills our students need to have when they leave high school. Have you ever wondered what the top 500 American companies indicate are the critical skills necessary for success? See Note A on next page.</i></p> 	<p>BLM 39: Transparency</p>  <p>The transparency shows the cover of Fortune magazine with the headline '100 BEST COMPANIES TO WORK FOR'. It features a cartoon character with a speech bubble that says 'HOW DOES YOURS STACK UP?'. The cover also includes a barcode and the text 'FORTUNE MAGAZINE COVER'.</p>

Activities

Part 2: Fortune 500 (continued)

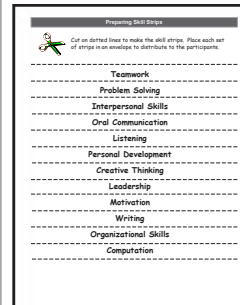
2. Group ranking:
 - a. Have participants sit in groups and distribute one envelope to each group with the 13 skill strips, that have been previously prepared according to the instructions on the **BLM 40: Preparing Skill Strips** handout for facilitator.
 - b. Have individuals choose what they think would be the top three skills.
 - c. Have groups reach consensus and place all thirteen skills in order from most important to least important.
 - d. Visit groups while they are working to answer questions and observe.
3. Processing and review:
 - a. Ask each table group for their top 3 ranking skills.
 - b. Write the responses on a blank overhead transparency.
 - c. Have the participants identify the skills ranked as most important by the entire group.
 - d. Distribute **BLM 41: Fortune 500 Companies**, handout of the ranking by the Fortune 500 Companies and display the transparency.
4. Ask the following discussion questions:
 - *If these are skills people need in order to be effective in the workplace, then when and how should they be learned?*
 - *Which of these skills should affect how mathematics is taught?*

Have the groups discuss the questions and then share their thoughts with the whole class.

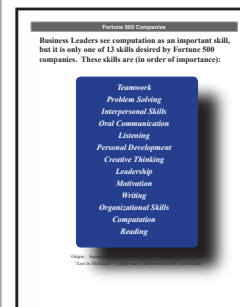
Notes

A. NOTE: The top 500 companies are according to Fortune magazine. To see a list of the Fortune 500 companies go to website www.fortune.com. The critical skills necessary for success are from an article in the Sacramento News & Review, Vol. 9, No. 2, 4/27/97.

BLM 40: Handout



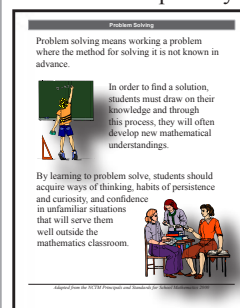
BLM 41: Transparency / Handout



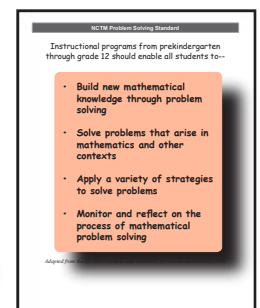
Part 3: Problem Solving (40 minutes)

1. Tell participants that problem solving is a very important part of mathematics. Today, problem solving is used in order to discover new ideas as well as to apply mathematical knowledge to new situations. Display the transparency of **BLM 42: Problem Solving** to highlight main ideas of problem solving from the NCTM standards. Distribute the **BLM 43: NCTM Problem Solving Standard** handout.
2. Tell participants that they are about to be given a problem but before they are given it, they need to think about assumptions.

BLM 42: Transparency



BLM 43: Handout



Activities

Part 3: Problem Solving (continued)

3. Display the **BLM 44: Assumptions** transparency, then discuss the use of assumptions in problem solving. Tell the participants that in order to solve a problem, they will have to make assumptions. Give an example of an assumption that is made on a problem:

How many fingers do we have in this room?

Share the poster that was made up ahead of time. Explain the logical thinking behind the assumptions. Point out that it was impossible to know exactly how many people would be present in the workshop and so this was the best that could be done with the information that you had.

4. Determine which of the following questions is appropriate for the session and use the corresponding transparency for **BLM 45** or **46** to present the question.

- a. For parent workshops use **BLM 45: Fill 'er up, Please!**:

How many gas stations are there in Tucson?

(Replace Tucson with your city)

- b. For family workshops with students 5th -8th grades use **BLM 46: That's a lot of Pizza!**:

How many pizzas are eaten by the people at your school in a year?

5. Have participants think about the question on their own for about five minutes and talk to their group about how they might answer this question. Let them know that there are calculators available for their use.

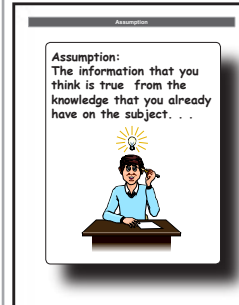
6. Display the transparency of **BLM 47: Poster Instructions** to create a team poster. Participants should work in groups of three or four to solve the problem and will develop a poster that:

- States their assumptions.
- Shows their methods.
- Gives their solution.

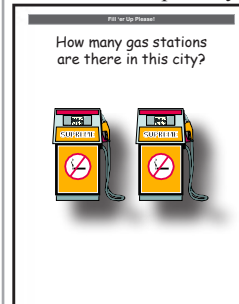
While the teams are working, be alert to what is happening, asking questions and directing the preparation of posters. For groups that finish early, an extension question for each problem is listed in the mathematics background. Give the groups a five or ten minutes warning to complete their posters.

Notes

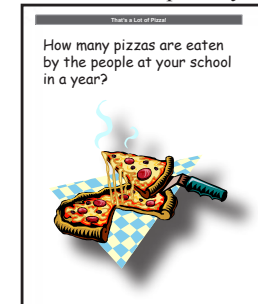
BLM 44: Transparency



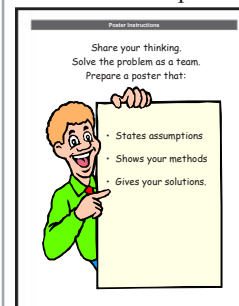
BLM 45: Transparency



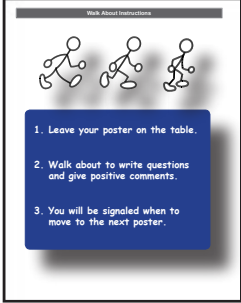
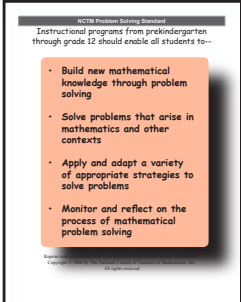
BLM 46: Transparency



BLM 47: Transparency



Activities

Part 4: Walk About and Discussion (30 minutes)	Notes
<p>1. Display the transparency of BLM 48: Walk About Instructions for the instructions for a "walk about". Participants need to have post-it notes and pens or pencils to write comments on each other's posters as they walk about.</p> <p>2. When the walk-about is complete, process it by asking the following questions:</p> <ul style="list-style-type: none"> • <i>What mathematics did you do in this problem?</i> • <i>What did you notice in the walk about?</i> • <i>What surprised you?</i> <p>The discussion should include:</p> <ol style="list-style-type: none"> Mathematical processes. Similarities and differences in approaches to the problem. 	<p>BLM 48: Transparency</p> 
Part 5: Connections (5 minutes)	
<p>1. Review the ideas that came out of Fortune 500. Ask participants which of these skills were important in the activity of the session.</p> <p>2. Revisit the standards on problem solving from the NTCM Standards by displaying the BLM 43: NCTM Problem Solving Standards transparency and leading a discussion on what occurred (in relation to the standards) while problem solving. These standards have been developed by thousands of math educators who have looked closely at the needs of today's students. Ask participants how these standards relate to the activity of the session.</p> <p>3. Let participants know that students are using or should be learning these skills in their classrooms. They will see this influence as they attend additional MAPPS sessions and while visiting their children's classrooms. This will help them have a better understanding of how and what their children are learning.</p> <p>4. Tell participants that this session's goal was to introduce them to:</p> <ol style="list-style-type: none"> The importance of seeing problems solved using different strategies. The connection between what employers value and how problem solving can be used in mathematics. 	<p>BLM 43: Transparency / Handout</p> 

Activities

Part 6: Take Home Applications (5 minutes)

1. Wrap up the session with an introduction to the handouts for parents.

BLM 49: Instructions for Puzzle for Home

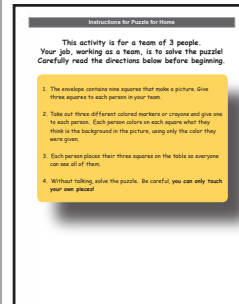
BLM 50: Puzzle for Home

BLM 51: Resources

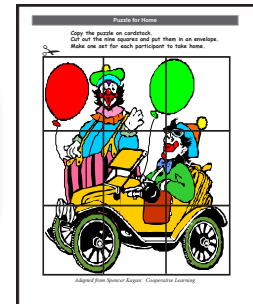
2. **BLMs 49-50** contains an enjoyable puzzle for working together with others. **BLM 51** is a handout of resources for problem solving ideas. You may want to show participants a copy of Figure This!. Computers with internet access are usually available at the local library, if not available at the school.

Notes

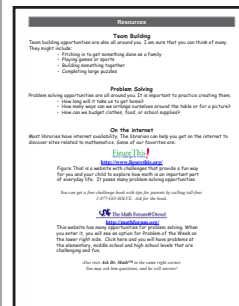
BLM 48: Handout



BLM 49: Handout



BLM 51: Handout



Part 7: Closing (5 minutes)

1. If your district does not have an evaluation form to use, have them answer one of the following questions:
 - Was the room quiet or did people talk to each other to learn mathematics?
 - Was the conversation "cheating"?
 - What tools did you use?
 - Did the facilitators tell the answers?

The discussion can center around the idea that noisy classrooms are not out of control classrooms, and that discussing problems can be very enlightening for students.

2. Distribute any prizes from estimations or drawings.