Only Five Regular Polyhedra

Activity 1. Counting the Faces, Edges, and Vertices of Regular Polyhedra

Why are there only five regular polyhedra?
At least three polygons are required to create a polyhedral vertex, and the number that can fit together at that vertex depends on the polygonal angle. The sum of the polygonal angles at any vertex must be less than 360º, because otherwise the configuration of polygons could not be folded up.

- If we use equilateral triangles we can have: 3 triangles, 4 triangles or 5 triangles meeting at a vertex (six triangles form a flat configuration, because the sum of the angles meeting at one vertex is 360º.
- If we use squares we can have 3 squares only meeting at a vertex (four squares form a flat configuration)
- If we use regular pentagons we can have only 3 meeting at a vertex (the sum of the angles of four pentagon meeting at a vertex is 4 x 108o, which is greater than 360º.
- We cannot use 3 hexagons, because they form a flat configuration.
- The sum of three angles of a regular heptagon or polygon with more than 6 sides is greater than 360º.
- So the only possibilities are 3 triangles, 4 triangles or 5 triangles meeting at a vertex (tetrahedron, octahedron, icosahedron); 3 squares meeting at a vertex (cube); or 3 regular pentagons meeting at a vertex (dodecahedron).

How to count the edges of a regular polyhedron
The icosahedron has 20 triangular faces, that is, each face is formed by 3 edges, but each edge is shared by 2 faces. So the number of edges is 20 x 3 / 2 = 30

The dodecahedron has 12 pentagonal faces, that is, each face is formed by 5 edges, but each edge is shared by 2 faces. The number of edges is 12 x 5 / 2 = 30