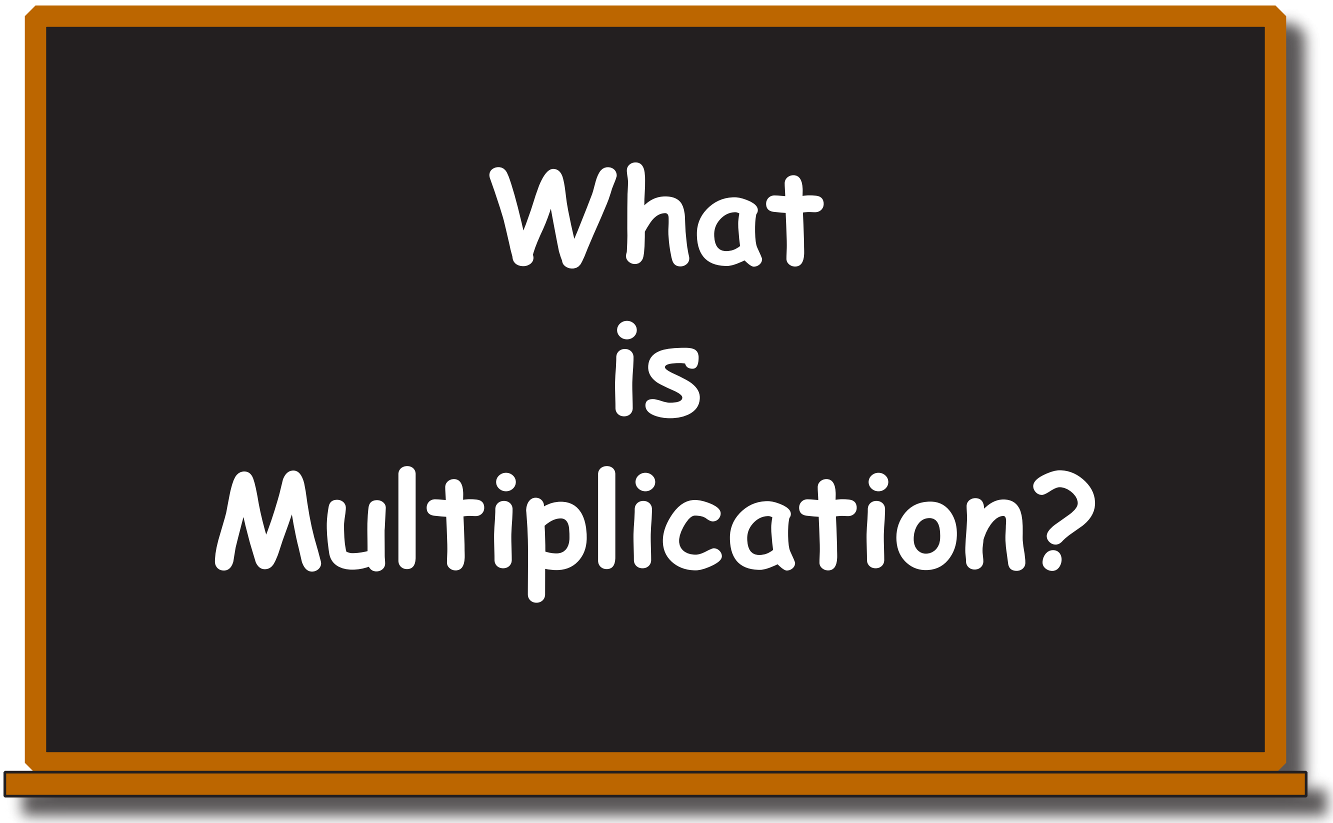
What is Multiplication?



One-Inch Graph Paper

Finding Multiples

**Numbers can be arranged into various numbers of rows as you can see.
Record numbers that can be arranged in the following number of rows:**

1	2	3	4	5	6	7	8	9	10	11	12	13

1. Did you find at least one number that could go under each list?
   If not, why do suppose that is?

2. Are there numbers that cannot be put in any of these columns?
   If there are, list those numbers.

3. What other things do you notice about these arrays of numbers?

List of Terms

What does it mean?

Array: One model for multiplication is an array. An array is a rectangle (remember that all squares are rectangles). To multiply 3 x 4, make a 3 by 4 rectangles and count the square units.

Composite Numbers: Composite numbers are numbers which have more than two whole number factors (4 is composite because its factors are 1, 2, and 4. The number 8 is composite because its factors are 1 ,2, 4, and 8). Composite numbers have more than one array. 4, 6, 8, and 9 are examples of composite numbers.

Even Numbers: Even numbers are numbers that are divisible by 2 with no remainder. Every even number forms an array that has a length and/or width of two units. 0, 2, 4, and 6 are examples of even numbers.

Factors: Factors are the numbers which are multiplied together to obtain the product (3 and 4 are factors of 12 because 3 x 4 = 12). The length and width of the arrays are the factors.

Multiple: A multiple of a number is the product of that number and any whole number (multiples of 3 are 0, 3, 6 ,9, 12,...), e.g., all the multiples of 3 have an array that has a length and/or width of 3.

Odd Numbers: Odd numbers have a remainder when divided by 2. They do not have arrays that have a length or width of 2. 1, 3, 5, 7 , and 9 are examples of odd numbers.

Prime Numbers: Prime numbers have exactly two factors, one and the number itself. One is not a prime number because it only has one factor, one. Numbers greater than one that have only two arrays are considered prime numbers. 2, 3, 5, 7, and 11 are examples of prime numbers.

NCTM Number and Operations Standard

Instructional programs from prekindergarten through grade 12 should enable all students to--

- ☐ **Understand numbers, ways of representing numbers, relationships among numbers, and number systems**
- ☐ **Understand meanings of operations and how they relate to one another**
- ☐ **Compute fluently and make reasonable estimates**

In thinking over the activities of this workshop, which of these standards have we worked with, and how did we work with that standard?

Rectangle Rodeo

Materials:

*One set of digit cards, 1 – 9, for each player
One game grid (**BLM 10**) for two players
Crayons or markers in two colors

Purpose:

Practice visualizing products as rectangular arrays.

Number of Players:

Two

Directions:

1. Shuffle all the digit cards (mix Player 1s digit cards with Player 2s digit cards) and place them face down to form a draw pile. Each player chooses his/her color marker or crayon.

2. Player 1 turns over two digit cards. He/she outlines a rectangle to represent the product of these two numbers. For example, if 4 and 6 are drawn, Player 1 outlines a 4 by 6 rectangle anywhere on the grid using his/her color. Player 1 also writes the multiplication problem and answer within the rectangle (4 x 6 = 24). Player 1 returns these digit cards to the deck.

3. Player 2 draws two digit cards and does the same, but may not overlap Player 1's rectangle.

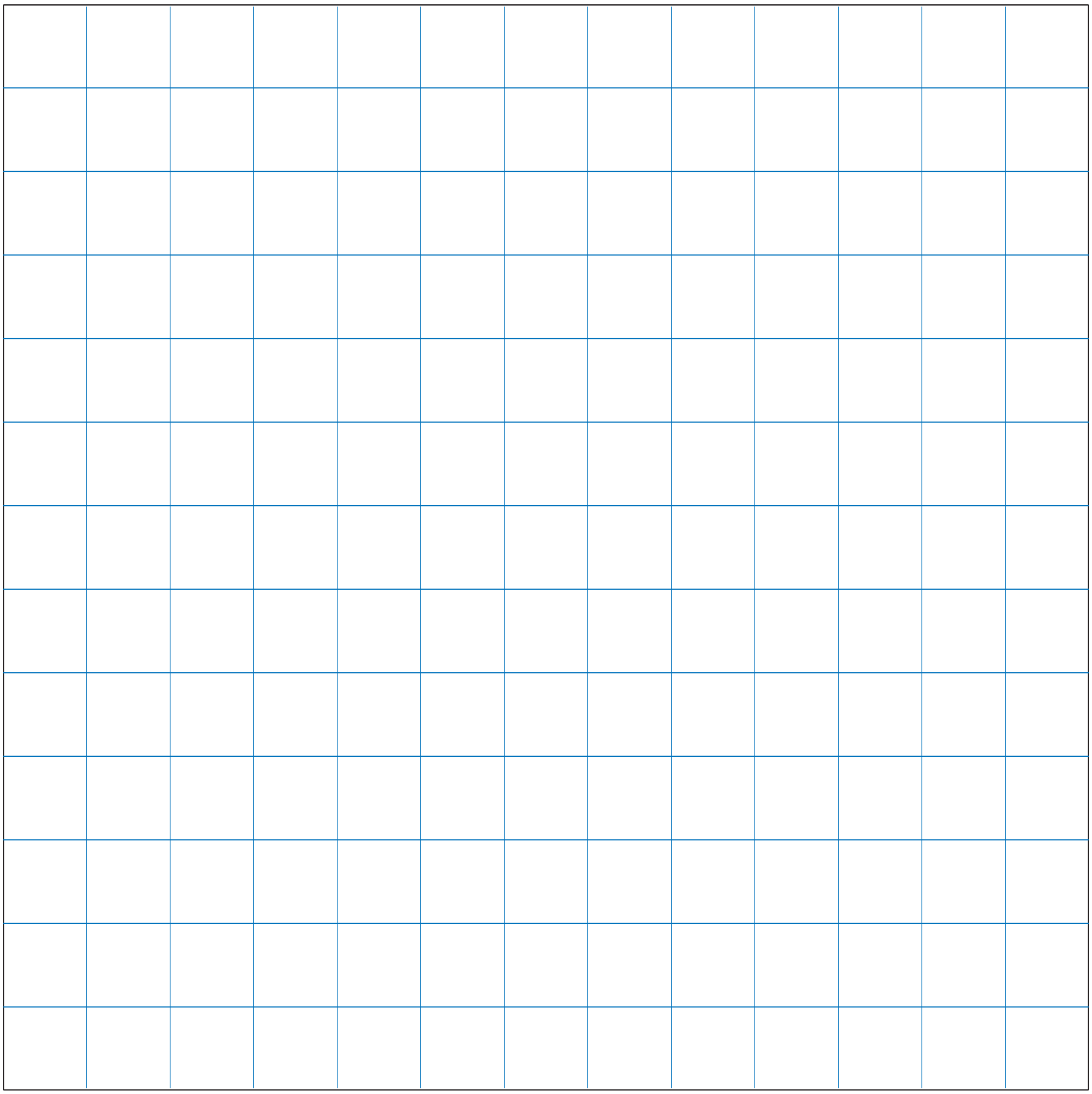
4. If at any time, a player has no place to outline a rectangle in the size he/she needs, play passes to the other player.

5. Play until no more rectangles can be drawn.

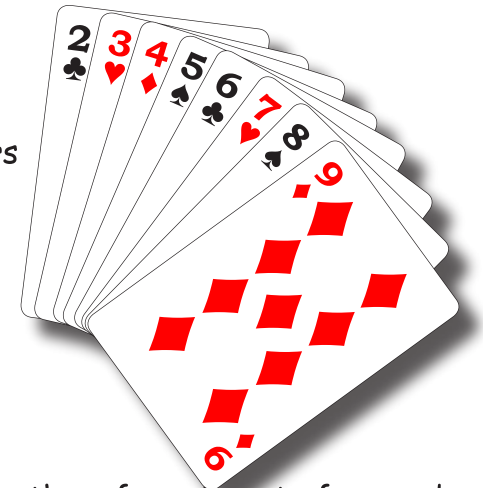
6. The winner is the player with the most rectangles outlined on the board.

*NOTE: These can be made from index cards or scraps of paper, or playing cards A – 9 may be used as digit cards (where A = 1).

Rectangle Rodeo Game Grid



Claim the Factors



Materials:
*One set of digit cards, 2 – 9, for each player
One set of product cards (**BLM 12**) for group of players

Purpose:
Practice naming factor pairs for products.

Number of Players:
Two - Four

Directions:

1. Shuffle product cards (small printed cards) and place them face down to form a draw pile. Each player places his/her digit cards face up in front of him/her.

2. Player 1 turns over a product card and states a pair of factors for this number. Player 1 claims his/her digit cards for these factors. For example, if the product card is 12, Player 1 could say, “3 x 4 = 12” and claim his/her 2 and 4 digit cards, or he/she could say “2 x 6 = 12” and claim his/her 2 and 6 digit cards. Only ONE pair of factor cards can be claimed on each turn.

3. If a product card shows a square number, ONE digit card may be claimed for that factor. For example, if the “25” card is drawn, the player may state “5 x 5 = 25” and claim his/her 5 card.

4. Each player, in turn, does the same. In subsequent rounds, a player may claim ONE digit card if the other factor has already been claimed. For example, if a player has already claimed his/her 3 and 4 digit cards then draws the product card 27, he/she may state “3 x 9 = 27” and claim the 9 digit card on this turn.

5. If a player already has all factors for the number drawn, play passes to the next player.

6. The winner is the first player to collect all of his/her digit cards, 2 - 9.

*NOTE: These can be made from index cards or scraps of paper, or playing cards 2 – 9 may be used as digit cards.

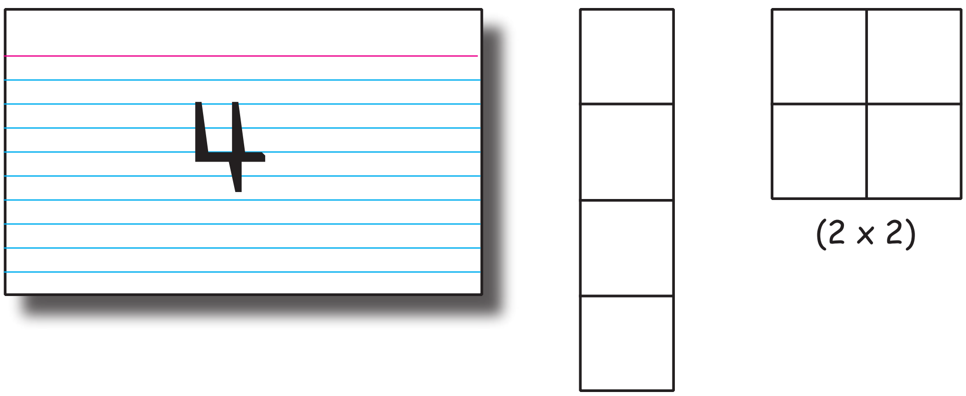
Claim the Factors Product Cards

12	14	16	18	20
15	25	30	35	40
36	45	54	63	72
21	24	27	28	32
42	48	49	64	72
6	8			

Activity for Home

"Making Arrays"

1. Cut one sheet of grid paper into individual squares.
2. Give your child the 3 x 5 index cards and have them write one number on each card between 3 and 36.
3. Pick one of the cards and have your child make all the possible arrangements of the number using the paper squares.
4. Help your child label their arrangements on a piece of paper (i.e. 2 x 3 or 3 x 2).



(1 x 4) (2 x 2)

Learning the Multiplication Facts

Learning the multiplication facts is a process that every student experiences. It is helpful for students to understand multiplication as an array, as in the activity tonight. It is also helpful for them to think of multiplication as repeated addition: 9 x 2 and 2 x 9 can be thought of as 2+2+2+2+2+2+2+2+2 or as 9+9, When looking for patterns with your child in the exercises below, be patient and let the patterns come from them. It is okay that they do not see them at first. Give it time and make it a fun time together.

Order of learning multiplication:

Multiplication by 2 (2 x 1 = 2, 2 x 2 = 4, 2 x 3= 6, 2 x 4 = 8, etc.)
The first group to practice is multiplication by 2. This group is easier for children because they already know that 9 + 9 is 18 from their addition facts. They need to connect multiplication by 2 to adding the same number to itself. Using something to count can help your child make this connection. Use squares, beans or something similar to show that 2 piles of 9 is the same as 2 x 9. Remember that 9 x 2 has the same result as 2 x 9. It is helpful to present both forms of the problem.

Multiplication by 10 (1 x 10 = 10, 2 x 10 = 20, 3 x 10 = 30, 4 x 10 = 40, etc.)
Many teachers do not test this, but it is a base for many mathematical concepts and is extremely important. Most students can count by 10s. To transfer this knowledge into the facts, give the students some problems, then look for patterns.
Patterns that they may notice:

1. All of the answers end in zero.
2. To get the answer, they place a zero beside the number they are multiplying because that number tells them how many tens they have.

Multiplication by 5 (5 x 1 = 5, 5 x 2 = 10, 5 x 3 = 15, 5 x 4 = 20, etc.)
Have your child work on multiplication by 5 next. This group is easier for children because they can count by fives. Set up piles of fives and have your child count them by counting by fives, e.g.. 5, 10, 15.... Have them record the answers and look for patterns to help them remember the answers.
Patterns that they may notice:

1. The answers are half of the same number multiplied by 10.
   Examples: 2 x 10 = 20, while 2 x 5 is half of that, or 10.
   4 x 10 = 40, while 4 x 5 is half of that, or 20.
   Why do you think this works this way?
2. All of the multiples of five end in five or zero.
   Which answers end in five, and which end in zero? Is there a pattern?

Multiplication by 9 (9 x 1 = 9, 9 x 2 = 18, 9 x 3 = 27, 9 x 4 = 36, etc.)
Multiplication by 9 is one of the most interesting patterns. Have your child develop the list of multiplication facts for 9 with arrays, beans or something that is hands-on. Then look at the list for patterns.
Patterns that they may notice:

1. All of the answers add up to nine when the two digits are added together:
   Examples: 2 x 9 = 18. 1 + 8 = 9 and 3 x 9 = 27. 2 + 7 = 9
2. The tens digit is one less than the number being multiplied by 9 because 4 times 9 is less than 4 times 10. Examples: **4** x 9 = **3**6, and 3 is one less than 4;
   9 x **7** = **6**3, and 6 is one less than 7

Learning the Multiplication Facts

Strategies for learning multiplication by 9

1. Use what you know about multiplying by 10 when multiplying by 9.
   Examples: 3 x 9 = 3 x 10 - 3, you subtract 3 from 30 and get 27.
   6 x 9 = 6 x 10 - 6, you subtract 6 from 60 and get 54.
2. There is also a hand method for this group. Place your hands flat on the table. If you are multiplying 6 x 9 (for instance), count to 6 starting from the left with your little finger and turn under your 6th finger (in this case the right thumb). Now that you have your finger turned under, count again from your left little finger, counting by tens until you get to the turned under finger: 10, 20, 30, 40, and 50. After the turned finger, count by ones: 1, 2, 3, 4. The answer is 54. Try it again for 4 x 9. Count to your 4th finger and turn it under. Then start again and count the first 3 fingers by 10s. Move to the other side of your turned finger and count by ones. This should be 6. The answer is 36.

Multiplication of numbers by themselves (1 x 1 = 1, 2 x 2 = 4, 3 x 3 = 9, 4 x 4 = 16, etc.)
Multiplying numbers by themselves result in square numbers, or perfect squares. This group just seems easier for children to learn.

The rest
After children have learned their 2s, 10s, 5s, 9s, and squares, there are ten facts left to learn, considering that when they learn 3 x 4 = 12, they have also learned that 4 x 3 = 12. These ten facts are 3 x 4 = 12, 3 x 6 = 18, 3 x 7 = 21, 3 x 8 = 24, 4 x 6 = 24, 4 x 7 = 28, 4 x 8 = 32, 6 x 7 = 42, 6 x 8 = 48, and 7 x 8 = 56.
Strategies for learning the additional ten facts:

1. For the 3s, think of 2 times the number and add on.
   Examples: 3 x 6 is 2 x 6 + 6. So it is 12 + 6 = 18.
   3 x 8 is 2 x 8 + 8. So it is 16 + 8 = 24.
2. For the 4s, think of doubling 2 times the number.
   Examples: 4 x 6 is 2 x 6, doubled. 12 + 12 = 24.
   4 x 7 is 2 x 7, doubled. 14 + 14 = 28.

Things to think about
Important concepts that come up again and again are doubling and knowing the sums to 10. An exercise to do with your children is to have them think of any number, then double it. If the number is small enough, you can build it together with beans, and then double the beans and count them all. To build the understanding of sums to ten, play *"How close am I to ten?"* Do this by using blocks or beans and taking out 6 and asking the question, *"What does 6 need to get to 10?"* A variation of this activity is to take out 7 beans and have your child take out 6 beans to hold. Ask your child to give you enough of their beans so you will have ten. Then ask them how many they have left and how many there are all together.