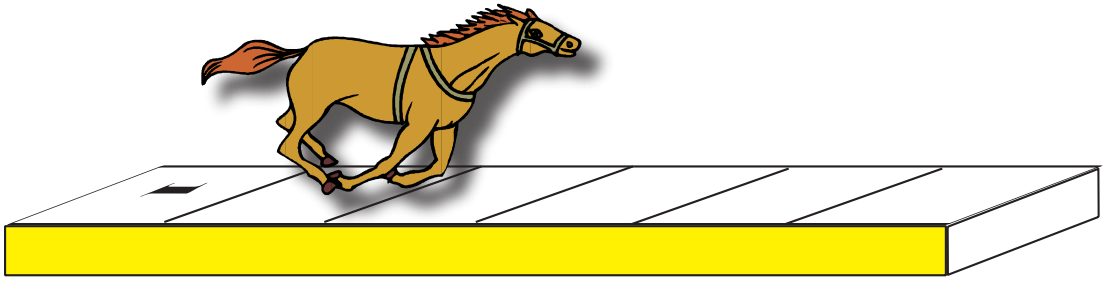
Probability



Chance and Predictions

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Outcomes

- To use ratios and a variety of vocabulary to describe the likelihood of an event.
- To use samples to make predictions.
- To provide parents an opportunity to explore probability and apply it to everyday life.

Overview

After participants are greeted, do an estimation and learn a little about the MAPPS program, they explore the likelihood of an event. They graph several events on a numberline numbered from 0 (Never) to 1 (Always). There is an opportunity for discussion about the choices that everyone has made, and participants get comfortable with the idea that a 0% chance means that something definitely will not occur, and that the probabilities run from there to 100%, when an event will certainly occur.

Next, a game using the sum of two number cubes is introduced and played. While having fun, the participants get an informal introduction to probability.

Using a matrix, participants explore the sample space for the game. They compute the probability of each sum being rolled. Through discussion, they connect the probabilities to the rolling of the number cubes.

The activity portion of the session is closed by exploring whether the original game is fair. Then they have the opportunity to examine a similar game to determine its fairness. This line of thinking is encouraged in the take-home activities as well.

If time permits, the participants can explore theoretical and experimemtal probability by looking at the diffences between their experiments and the theoretical probability that was developed through the matrix.

At this point facilitators have the opportunity to bring in ideas from the district curriculum. This is also a time to make connections among the topics of this session, the curriculum samples, and the national mathematics standards on probability.

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Mathematics Background

The mathematical concepts in this module include:

- Probability: Sample space (possible outcomes)
  a) Probability of an event (See Note C of facilitator notes)
  b) Probability notation (See page 8 of facilitator notes)
- Number Sense: Creating fractions to represent an event (See **BLM 68: Probability**) Ratios

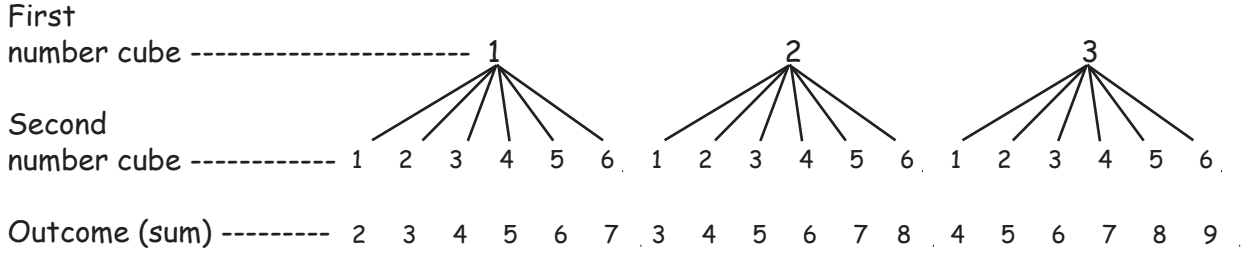
Theoretical and experimental probability

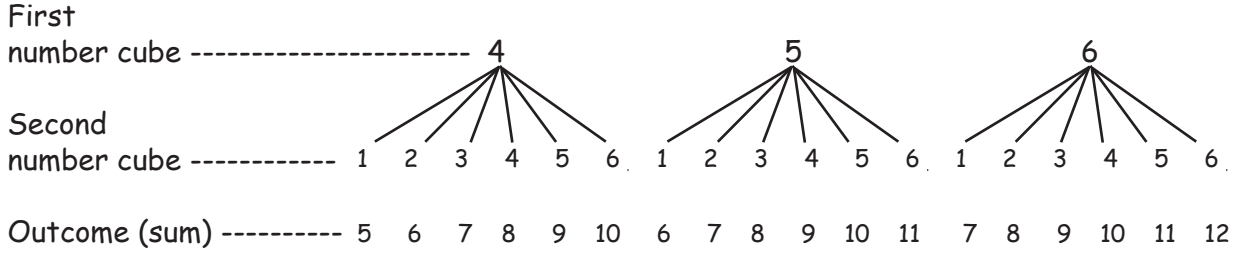
An extension to this workshop is the exploration of theoretical versus experimental probability. Experimental probabilities are found through simulations or by experiments. Simulations are situations that can act the same way as the actual experiment. The races at the beginning of the module are a simulation that can be used for experimental probability. Find the total number of races, and total the number of times that each horse won. These are the experimental probabilities of each horse. Convert them into percentages by dividing the numerator by the denominator. For example, if there were 40 races, and number 7 won 10 times, P(7) = 10/40. 10 divided by 40 = .25 or 25%. So the experimental probability of 7 is represented as P(7) = 25%.

The theoretical probability was calculated in the outcome matrix. It shows that, theoretically, 7 should win 6 out of 36 times, or a little less than 17% of the time. Why don't the theoretical and experimental probabilities match? The idea behind theoretical probability is that in the long run, the experimental probability will get closer and closer to matching the theoretical probability.

Sample Space

The sample space of an event is a listing of all possible outcomes. The common ways for doing a sample space are a matrix, as used in this workshop, a tree diagram (illustrated below), or simple inspection (as in a single number cube). Care must be taken to include both 1 and 6 making 7 and 6 then 1 making 7. These a separate sample spaces, and experimental probability can be used to verify the idea.





Ratios

Ratios can be expressed in many different forms. The form that is used in this module for probability is the fraction form (25/100). They can also be represented by percents (25%), decimals (.25), and comparisons (25:100, read as "25 to 100").

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Room Setup

- Desks or tables arranged in groups of 4
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda
- Space to hang butcher paper
- Space for participants to line up

Materials

Facilitator	Transparencies
• Overhead projector • Overhead pens • Overhead square tiles • Transparencies, write-on • Chart paper • Chart markers • Masking tape • Timer (optional) • Butcher paper • Estimation questions (prepared by facilitator) • Inexpensive prizes	*BLM 1: Welcome* *BLM 64: Never to Always Scale* *BLM 65: The Horse Race Directions* *BLM 67: Outcome Matrix* *BLM 69: Probability Directions* *BLM 71: NCTM Expectations*
Participant	**Handouts**
Individuals • Paper • Pencil • Reflection • Calculators **Groups of 2-4** • Horse Race gameboards • Two number cubes (or dice)	**One per participant for class** *BLM 63: Never to Always* *BLM 67: Outcome Matrix* *BLM 68: Probability* *BLM 70: Is It Fair?* *BLM 71: NCTM Expectations* **One per group for class** *BLM 65: The Horse Race Directions* *BLM 66: The Horse Race Gameboards* **One per participant for home** *BLM 72: A Different Race* *BLM 73: Game Variations for Home* *BLM 74: Double Trouble*

Timing

2 hours

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Preparation and Timing (1 hour and 55 minutes)

Part 1: Getting Started (10 minutes)

Display transparency from workshop one:
BLM 1: Welcome
Make a copy for each participant:
BLM 63: Never to Always

Part 2: Setting the Stage (10 minutes)

Make transparencies of:
BLM 64: Never to Always Scale

Part 3: The Horse Race (15 minutes)

Make transparencies of:
BLM 65: The Horse Race Directions
Make a copy for each group:
BLM 65: The Horse Race Directions
BLM 66: The Horse Race Gameboards

Part 4: Computing Probability (30 minutes)

Make transparencies of:
BLM 67: Outcome Matrix
BLM 69: Probability Directions
Make a copy for each participant:
BLM 67: Outcome Matrix
BLM 68: Probability

Part 5: Creating a Fair Game (30 minutes)

Make a copy for each participant:
BLM 70: Is It Fair?

Part 6: Connections (10 minutes)

Make transparencies of:
BLM 71: NCTM Expectations
Make a copy for each participant:
BLM 71: NCTM Expectations

Part 7: Take Home Applications (5 minutes)

Make a copy for each participant:
BLM 72: A Different Race
BLM 73: Game Variations for Home
BLM 74: Double Trouble

Part 8: Closing (5 minutes)

No handouts or transparencies
3-4 Inexpensive prizes for Estimation Question winners
Reflection / evaluations *(provided by the evaluation team)*

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Facilitator Resources

Articles

Aspinwall, Leslie and Shaw, Kenneth, "*Enriching Students' Mathematical Intuitions with Probability Games and Tree Diagrams"*, Mathematics Teaching in the Middle School, Vol. 6, No.4, December 2000. p. 214.

Books

Math on Call: A Mathematics Handbook. Great Source Education Group, 1998, p. 465.

They're Off! by Alfinio Flores

Mathematical Activities from Poland by Jerzy Gwirko-Goadycki

Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, P. 176, ISBN 0-87353-480-8, www.nctm.org

Instructional Programs

Connected Mathematics Program: *How Likely Is It?*, Lappan, Fey, Fitzgerald, Friel, and Phillips, Prentice Hall, 2002.

Internet Connections (site addresses updated 5/29/03)

Project Interactive has several interesting activities. Their home page is:
http://www.shodor.org/interactivate/index.html
A few examples of their probability activities are given below:
Interactive Car Race: 1 die, you have capability to change the game
http://www.shodor.org/interactivate/activities/race/index.html
Monty Hall's door: switch or stay with first choice
http://www.shodor.org/interactivate/activities/monty3/index.html

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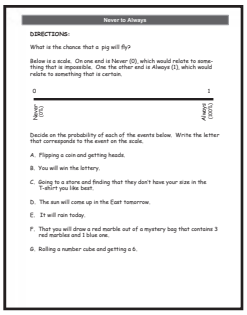
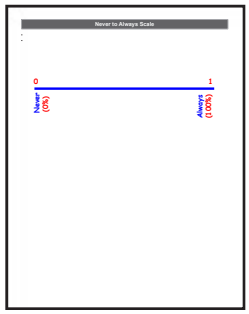
Activities

Preparation of Classroom	Notes
1. Arrange desks or tables in groups of 4-6. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up three or four estimation activities. Have **BLM 63: Never to Always** on the tables for participants to discuss as they arrive. 2. Display the transparency of **BLM 1: Welcome!**. 3. Prepare and display a poster with the agenda and purpose of the session.	BLM 1: Transparency Welcome MAPPS MATH AND PARENT PARTNERSHIPS Math Awareness Workshop WELCOME! Please do the following: 1. Sign in and complete any necessary paper work. 2. Do the estimation activity located on the table by the door. 3. Help yourself to refreshments and enjoy. 4. Please find a seat and wear your name tag. BLM 63: Handout  Never to Always DIRECTIONS: What is the chance that a pig will fly? Below is a scale. On one end is Never (0), which would relate to something that is impossible. One the other end is Always (1), which would relate to something that is certain. 0 1 Never (0%) Always (100%) Decide on the probability of each of the events below. Write the letter that corresponds to the event on the scale. A. Flipping a coin and getting heads. B. You will win the lottery. C. Going to a store and finding that they don't have your size in the T-shirt you like best. D. The sun will come up in the East tomorrow. E. It will rain today. F. That you will draw a red marble out of a mystery bag that contains 3 red marbles and 1 blue one. G. Rolling a number cube and getting a 6.
Part 1: Getting Started (10 minutes)	
Introductions 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences. 3. Give participants an overview of the session. Review the agenda and purpose of the session. When discussing the agenda, let the participants know the plan for including children in the session.	
Part 2: Setting the Stage (10 minutes)	
1. Participants discuss their placement of the items on Never to Always. Facilitator records (or has the participants record) some of the events on a transparency of **BLM 64: Never to Always Scale**. Other vocabulary words that might arise from this discussion are: even chance, rare, unlikely, sometimes, certain, more likely, and probably. If these come up naturally, discuss them. 2. Ask where probability is used in daily life. Record the ideas that are shared. Tell participants that the topic of this session is probability. They will play a game, and then learn how to compute probabilities. Finally, probability in the curriculum is discussed.	BLM 64: Transparency  Never to Always Scale 0 1 Never (0%) Always (100%)

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Activities

Part 3: The Horse Race (15 minutes)

1. Distribute two number cubes (or dice) and **BLM 65: The Horse Race Directions** to each group . Model playing the Horse Race game using the transparency of **BLM 65**.

2. Have participants form pairs or small groups. Have each group play 3 or 4 games of the Horse Race at their tables using **BLM 66: The Horse Race Gameboards**. Before processing, collect the number cubes. Record the winning sums from each game on chart paper. **See note A**.

3. Discuss the game. Remember that when participants notice things about the game, there are no wrong answers, just observations. Some of the ideas that should come up in the discussion are:
   a) The fact that the sum of 1 will never be rolled
   b) The sums of 6, 7, and 8 are the most frequently rolled
   c) Some sums are rare to roll, and that the game isn't fair (In a fair game, every player has an equal chance to win).

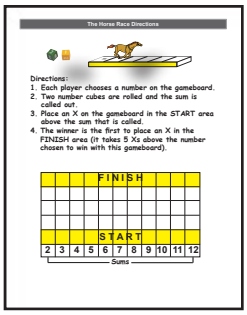
In order to draw these ideas from the participants, start with the following question:

What have you noticed about the game?

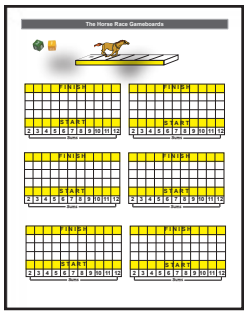
Other questions to ask to bring out the ideas above if they have not been mentioned are:

- *Are there any sums that come up more often or less often than others?*
- *Do you think that every number has an equally likely chance to win?*

BLM 65: Transparency / Handout



BLM 66: Handout



A. Note: Example of recording winning sums

1		7	𝍸
2	||	8	𝍸
3		9	||
4	|	10	|||
5	||	11	
6	𝍸	12	

Part 4: Computing Probability (30 minutes)

1. Tell participants that we are going to look at the mathematics that support our ideas about the game. Distribute **BLM 67: Outcome Matrix**, while using the transparency to illustrate filling in the matrix. **See Note B** for the filled matrix. Ask participants:

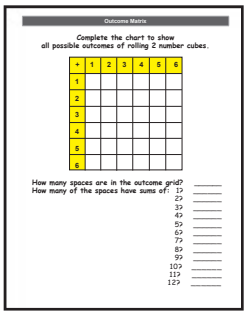
   If we rolled a one with our first die, and then a one with our second die, what would our sum be?

(Write the 2 in the box, modeling the total of that roll.) If needed, continue with examples of rolls.

2. Participants fill in the matrix and answer the questions under the matrix.

3. Ask:
   - *As we look at the occurences of each sum, are there any surprises?*

BLM 67: Transparency / Handout



B. Note: Filled Matrix

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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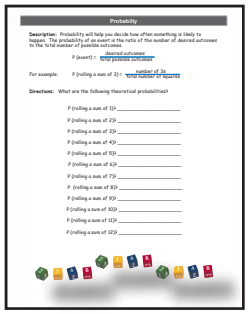
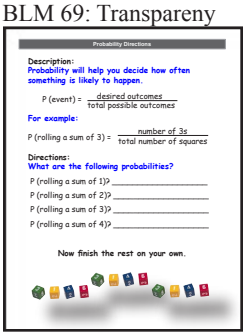
Activities

Part 4: Computing Probability (continued)	Notes
• *How does this information relate to the results on our game boards?* Participants should notice that the 6, 7, and 8 occur more often. • *Why do you think that these sums occur more often?* They should notice that there are more combinations for rolling these numbers. 4. Distribute **BLM 68: Probability** and display **BLM 69: Probability Directions** on the overhead. Explain the notations of P (rolling a sum of 3) = ___. This notation is read as "the probability of rolling a sum of 3 is ___." Model finding the probability of rolling a sum of 3 on a pair of dice. Refer back to the last sheet. Participants should notice and have recorded that the total number of possible outcomes for rolling two dice is 36. **See Note C**. Some participants will want to reduce this to 1/18. It is best not to have this happen at this time (participants will later add all their probabilities). 5. Have participants complete Probability. **See Note D**. Ask: • *What is meant by P(rolling a 7) = 6/36? Approximately how often could one expect to roll a 7?* • *Where would we put this on the never to always line?* • *What is meant by the 0/36?* • *Where would we put this on the never to always line?* • *Which is more likely: rolling a sum of 10 or rolling a sum of 5? How do you know?* • *Find 2 sums that are equally likely to be rolled.* • *What do you think would happen if we added all the prob abilities on the probability table?* Have participants do this. They will discover that the sum is 36/36, or one whole. *How does this sum of one relate to our never to always line?* Relating back to the never to always, one meant that it would always occur. It means that a sum of 2 - 12 will always be rolled. *What do you notice about our list?* Ideas may include: each of the denominators is 36, the numerators go from small to large to small again, and the fractions are all less than one (or less than 1/2).	BLM 68: Handout  BLM 69: Transpareny  **C. Note:** $\frac{\text{\# of sums of 3}}{\text{total possible outcomes}} = \frac{2}{36}$ **D. Note:** P (rolling a sum of 1)? 0/36 P (rolling a sum of 2)? 1/36 P (rolling a sum of 3)? 2/36 P (rolling a sum of 4)? 3/36 P (rolling a sum of 5)? 4/36 P (rolling a sum of 6)? 5/36 P (rolling a sum of 7)? 6/36 P (rolling a sum of 8)? 5/36 P (rolling a sum of 9)? 4/36 P (rolling a sum of 10)? 3/36 P (rolling a sum of 11)? 2/36 P (rolling a sum of 12)? 1/36

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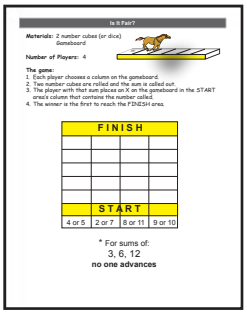
Activities

Part 5: Creating a Fair Game (30 minutes)	Notes
1. Ask: *Does everyone have a equal chance to win in the horse race?* Participants should now realize that there are some numbers that have a better chance than others. Explain that in a fair game, the probability of winning is the same for every player. Ask: *If there are only 2 horses in a race, #2 and #12, is this a fair game?* The participants should see that the probabilities are equal, so it is a fair game. Suggest to the group that sometimes players can each have more than one horse in their track. For instance, let's say that one player has the 2 and the 3 horses on their track, and they get to move forward when either is rolled. Say: *Let's look at a new horse race and see if it is fair.* 2. Distribute **BLM 70: Is It Fair?**. Have participants experiment with it to find out if it is fair. Some will evaluate it mathematically, using probabilities. Others will want to play it. They will need number cubes (or dice). Before processing, collect the number cubes (or dice). After 10 minutes stop everyone and have them share their ideas. Record ideas on an overhead transparency. They should have discovered that this is a fair game, because theoretically, each player has a combined probability of 7/36. Those who played may not be as clear about the fairness of the game. In the long run, it should be fair, but with each individual game, the results vary. 3. If time permits, explore the difference between experimental and theoretical probability. Have participants look at all of the rolls that they have made for the sum of the dice and make a chart of how many times each number has been rolled. Figure out how many total rolls everyone has done. Then have the participants compute the experimental probabilities of each sum by dividing the number of occurences for each number by total rolls. *Why are the experimental and theoretical probability different?* Have participants share their ideas. The experimental is subject to chance, and therefore will vary from the theoretical. However, as more and more rolls are made, the experimental will more closely reflect the theoretical probability.	BLM 70: Handout  Is It Fair? Materials: 2 number cubes (or dice) Gameboard Number of Players: 4 The game: 1. Each player chooses a column on the gameboard. 2. Two number cubes are rolled and the sum is called out. 3. The player with that sum places an X on the gameboard in the START area's column that contains the number called. 4. The winner is the first to reach the FINISH area. FINISH START 4 or 5 | 2 or 7 | 8 or 11 | 9 or 10 * For sums of: 3, 6, 12 no one advances

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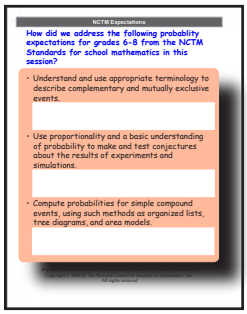
Activities

Part 6: Connections (10 minutes)

1. Discuss what probability looks like in today's classroom. It is important at this time to bring in ideas from your district's curriculum that relate to probability.

2. Display the transparency of **BLM 71: NCTM Expectations** to make connections among the topics of this session, your curriculum samples, and the national mathematics standards on probability topics. Distribute the handout of **BLM 71: NCTM Expectations** and have participants discuss in groups on how the expectations were addressed. Have groups share their ideas.

BLM 71: Transparency / Handout



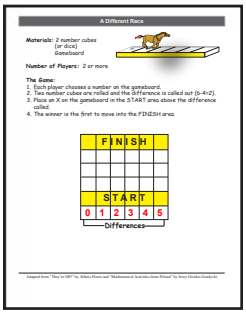
Part 7: Take Home Applications (10 minutes)

1. Suggest to the participants that they change the horse race so that the two number cubes are subtracted instead of added. If a 6 and a 4 are rolled, the player with the 2 would put an x on the gameboard above the 2. The gameboard has a 0, 1, 2, 3, 4, and 5. **BLM 72: A Different Race** has the instructions and a gameboard for this game.

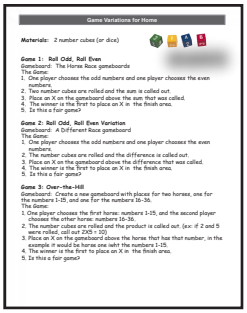
2. Distribute **BLM 73: Game Variations For Home** and **BLM 74: Double Trouble** to participants to do at home. These are activities using probability. If desired, participants can look at their matrix to decide on the probability of getting doubles. They can talk about what might be a good strategy for playing Double Trouble based on this probability.

3. Participants can look for ways that probability is used in daily life.

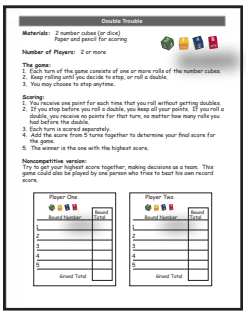
BLM 72: Handout



BLM 73: Handout



BLM 74: Handout



Part 8: Closing (5 minutes)

1. Distribute any prizes for Estimation Question winners.

2. If your district does not have an evaluation form to use, you may want to use a reflection similar to:
   - *What did you learn tonight?*
   - *What will you do with your child as a result of this session?*

3. Thank participants for coming and spending time doing mathematics despite their busy schedules.

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